Forecasting Ammunition Demand on a Modern Battlefield

A Thesis

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Abstract

Your favorite major retail store (online or brick and mortar) currently has the ability to forecast demand for commodities such as toothpaste and toilet paper. They use these forecasts to make decisions about the best quantity of goods to deliver to distribution centers and retail outlets and when to deliver them. They minimize shipping and storage costs and avoid having too many or too few items on hand at the point of need. Military units on today's battlefields would benefit greatly from a similar scheme to forecast ammunition requirements and make sound delivery decisions. The forecasting is more difficult though, as ammunition demand is often the result of external, unpredictable enemy decisions. Production is slow and requires long lead times. Delivery is dangerous and unpredictable. Available data is often incomplete or seemingly irrelevant. Even though the system that controls the need for ammunition is hidden from our direct observation (friendly and enemy unit actions and counteractions), we can still predict future system behavior through the analysis of time series data.

This research explores several popular forecasting methods to determine their strengths, weaknesses, and overall applicability to predicting ammunition demand by US Army units in Afghanistan from 2010 to 2013. Autoregressive integrated moving average (ARIMA), Exponential Smoothing, Hidden Markov Models, and historic average estimation models are all used to predict future ammunition demand. The resultant forecasts may be used to feed a comprehensive logistics planning system to help military leaders make informed decisions about commodity delivery on the battlefield in order to decrease risk and increase reliability of logistics resupply. Results indicate that forecasting the outputs of a system as unpredictable as war is very challenging. The univariate exponential smoothing models forecast with the least percent error for near term forecast horizons, and their accuracy is shown to improve with bootstrap forecast aggregation. A novel alteration to residual bootstrap aggregation is presented that increases forecast accuracy by mitigating the large variance for such a stochastic time series as ammunition demand. This research is relevant not only for military sustainment planners, but for anyone who works with demand that varies over time across several echelons of product and consumer.

Approval Sheet

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1 Introduction

"A Soldier in combat can go a year without pay; months without mail; days without food, water, and sleep; but he cannot survive one minute without ammunition."

(LTG Joseph M. Heiser, HELFAST XVII Conference, Fall, 1986)

Military leaders have identified a need to better estimate ammunition consumption on the modern battlefield. This problem has been made more complicated in recent conflicts because ammunition is often expended as a result of largely unpredictable enemy decisions and the will of the local populace as opposed to thoroughly planned friendly unit decisions. This inherent unpredictability coupled with high risks in overestimating or underestimating ammunition requirements makes this an urgent problem.[1] The goal of this research is to identify methods to improve the US military's ability to forecast ammunition consumption in complex combat scenarios through the time-series analysis of ammunition transaction data. This research seeks to take advantage of the inherent hierarchical structure of military units and ammunition types to increase forecast accuracy for a commodity with such an intermittent demand. A reliable forecast will feed a comprehensive logistics planning system to help military leaders make informed decisions about commodity delivery on the battlefield. This will result in decreased risk and increased reliability of logistics resupply.

Currently in US Military operations in Afghanistan, units are organized by space. An Army division may have responsibility for a region consisting of several provinces. The division's subordinate brigades (three to five brigades per division) may each have responsibility for one or more provinces including one or more large military bases that serve as distribution hubs. Each brigade's subordinate battalions (three to five battalions per brigade) may have responsibility for a single province or several districts of a province and be headquartered at a large Forward Operating Base (FOB). The battalion, consisting of three to five subordinate companies, will have its forces stationed at several smaller FOBs and Combat Outposts (COPs) in order to most effectively accomplish their assigned missions (See Figure 1 and Table 1 for more information). This organization by space makes the distribution of supplies straightforward. A sustainment unit at the division level is responsible for distributing supplies to hubs at the subordinate brigade locations. Brigade sustainment units distribute to battalion FOBs, and battalion sustainment units distribute to their subordinate units

[1]When ammunition needs are underestimated a military force may be incapable of defense in the face of an enemy force. Overestimation may place supply convoys in unnecessarily risky situations.

at smaller FOBs and COPs. This organized distribution scheme is not always followed exactly as the restrictive terrain in Afghanistan makes air movement necessary for many supply distributions and air assets are generally tasked by echelons higher than those who use their services [1]. This organized hierarchical scheme does, however, provide a convenient and relevant framework for our forecasting and distribution problem and so will be referenced throughout this research.

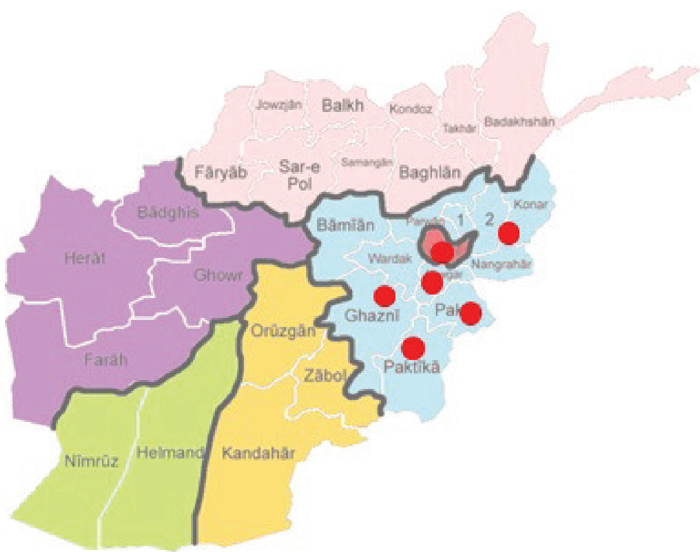

Figure 1: Afghanistan is divided into regional commands by the International Security Assistance Force (ISAF). US responsibility is largest in Regional Command East (RC East), and area about the size of Virginia. There are 11 provinces and the capital province of Kabul in RC East (these provinces are shown in light blue, Kabul is the red province in the center of RC East). There are six major Forward Operating Bases (FOBs) that serve as supply distribution hubs in RC East; they are depicted by red dots. These large FOBs also serve as headquarters locations for the six Brigade Combat Teams (BCTs) currently in RC East. There are hundreds of smaller FOBs and Combat Outposts (COPs) not shown in the figure out of which US forces work to accomplish their assigned missions.

Light Infantry Organization

Unit	Weapons	Personnel
Squad	Rifles, Grenade launcher, Machine Gun	10 Soldiers
Platoon (3-5 Squads)		40 Soldiers
Company (3-5 Platoons)	Mortars	150 Soldiers
Battalion (3-5 Companies)	Automatic Grenade Launchers, Missiles and Rockets	700 Soldiers
Brigade (3-5 Battalions)	Artillery	4,000 Soldiers
Division (3-5 Brigades)		15,000 Soldiers

Table 1: Typical organization of US Army light infantry units

Rudimentary demand forecasting is conducted at most levels along the distribution chain. These forecasts, however, tend to be very inaccurate and sometimes even unusable because they grossly under or overestimate actual need [2]. They tend to take two common forms: a forecast based on long-term historical data (an infantry battalion conducting offensive operations in desert terrain has been known to consume X rounds of machine gun ammunition per day) using the OPLOG Planner[2], or a naive forecast that estimates that next period's demand will equal the current period's demand. Supply requests are forwarded up from smaller units to the units with the requested material. In the case of ammunition, most units maintain their own small storage area in order to avoid ever being dangerously low on supply. Most ammunition requests, therefore are made in order to replenish supplies on hand. This procedure puts lags into the system between ammunition use, ammunition need, and ammunition delivery. The inner workings of this system are hidden to us: we don't know how much ammunition each unit keeps on hand, we don't know when ammunition gets used or how much gets used, we don't even know when ammunition gets requested. We do know when ammunition gets issued, what type gets issued, and how much gets issued. This information will form the basis of a forecasting strategy to help decision makers at all levels predict more accurately the ammunition to be delivered to subordinate units.

The next section will review relevant literature on military sustainment and demand forecasting

[2]The Army's Operations Logistics (OPLOG) Planner Software tool is a digital version of resource consumption estimation charts. It is currently used to estimate several commodities including fuel, water, and food. The ammunition estimates, however, tend not to be acceptably accurate in current complex combat scenarios as they were designed using data from previous, more linear conflicts.

methods applicable to our problem. Section 3 will further describe the specific attributes of our problem and lay out objectives necessary to overcome it. Section 4 will provide a discussion on the available data and detail our methodology to assist planners and decision-makers to better forecast ammunition needs. Results from our analysis will be discussed in Section 5; and Section 6 will conclude with the contributions of our methodology and recommendations for implementation and further research.

2 Literature Review

2.1 Military Sustainment

Much work has been done to identify ways to improve the flow of military logistics. RAND, in particular, has completed several studies [3, 4, 5] that illustrate ways to incorporate into military logistics methods that have already seen success in industry, such as just-in-time delivery and distribution. These reports demonstrate the need for a reliable ammunition estimate, but do not address how to make one. These studies also lack many of the insights gained during the Global War on Terrorism where asymmetric warfare on a non-contiguous battlefield presents a different set of challenges and improvised solutions to ammunition forecasting and delivery. More recently, in his Command and General Staff College Thesis, William Freeman pointed out the overestimation tendency of currently used military references for ammunition estimation and suggested a forecasting framework based on more recent historical data [6].

Army Field Manuals (FMs) [7, 8, 9, 10, 11, 12, 13] and studies published by the Center for Army Lessons Learned (CALL) provide commentary on current techniques used by Army units to estimate ammunition consumption and efficiently deliver ammunition on a battlefield. FM 101-10-1/1 and /2 were replaced by the OPLOG Planner Tool, but still make up its logic for forecasting. These FMs provide data about the composition of units (how many guns and vehicles they own), and planning factors to estimate consumption of commodities like food, fuel, and ammunition. FM 4-30.1: Munitions Distribution in the Theater of Operation details the process by which ammunition moves through a supply chain. It acknowledges the importance of ammunition forecasts to ensure the success of Army units, but only offers forecasting techniques for very simple and practical problems, such as to ensure the necessary haul assets and material handling equipment are available for an expected ammunition delivery. It also provides details on calculating a required supply rate for units based on their known upcoming missions based on references such as FM 101-10-1/2 and OPLOG Planner. The manual covers detailed methods for forecasting training ammunition which can be very accurately determined because unit size (and hence number of weapons) is known and training tasks are planned months in advance. The largest flaw in this manual is its suggestion that forecasting be conducted at the lowest possible level and only consolidated from there up. This method likely results in the overestimation explained in [6], and prevents decision makers and planners at higher unit levels from seeing an accurate forecast with a large enough time horizon to make necessary plans. The CALL publications mention various ways to improve convoy operations [14], patrols [15], ammunition distribution [16], and overall unit preparedness by declaring the importance of an accurate and timely forecast of ammunition use, but provide no information on how to make such a forecast.

These military publications provide a good contextual reference to the problem and help describe the limits of a possible solution, but they do not offer an accurate, usable estimation procedure. They are used in this research as guides to establish a problem solving framework that may be actually implemented in the near future and that will solve the right problem.

2.2 Time-Series Forecasting

Myriad time series forecasting methods have been shown to improve over naive methods where a future demand is of interest. Such methods include time series regression, moving average (MA) and auto-regressive moving average (ARMA) models, seasonal and non-seasonal auto-regressive integrated moving average (ARIMA) models, and generalized auto-regressive conditionally heteroskedastic (GARCH) models [17]. Because of the seasonal effects and dynamic nature of a battlefield, this research will focus on what we consider to be the most appropriate and adaptable methods: Exponential Smoothing and seasonal ARIMA methods. These two methods may be adapted (by changing parameters) to fit appropriately to any order of time-series [18].

The Exponential Smoothing method uses moving averages that are weighted exponentially, with higher weights assigned to the more recent averages. These weighted averages provide updates to the seasonally-adjusted mean, trend and seasonal components of the time series [19, 17, 20]. Because it includes the effects of trend and seasonality, and its responsiveness is adjustable for different situations, the Exponential Smoothing model is well suited for our demand forecasting problem [21, 22]. The method is largely attributable to the work of Holt (1957) and Winters (1960) [17].

ARIMA models are used to forecast non-stationary time series because they can capture the correlations of a series to itself through differencing. They also integrate a moving average term. And when they are extended to include seasonal terms they are capable of accurately modeling a broad array of series[19]. The ARIMA methodology is due to the work of Box and Jenkins in the 1970s [17].

When ammunition demand is considered at a low level, say, at battalion level for one type of ammunition, it forms a very intermittent time series. There are several time periods with no transactions followed by one big transaction. This is similar to the time series of gasoline purchased for an automobile: several days without buying gas, then one day when 20 gallons are purchased. J. D. Croston proposed a technique to better forecast series like these in 1972 by using separate estimates for both the size of the demand and its frequency [23]. Since then, several researchers have sought to improve or further explain his work. Their research has shown, however, that strict assumptions must be maintained for the technique to be effective. The demand process must be shown to be Poisson [24] and the underlying residual process must always be non-stationary [25]. The technique is also not robust to missing data, as one missing non-zero value may greatly affect the forecast.

2.3 State-Space and Hidden Markov Models

Other researchers have sought to better explain processes like ours, especially intermittent processes, through state-space models which may result in more accurate forecasts than traditional time-series analysis without completely explaining the state of the system at any given point in time [26, 27, 17]. State-space methods are worth exploring in order to make sense of a process for which we only have reliable data for one aspect (how much ammunition changes hands) and we are left to assume the rest (how much ammunition is actually needed, when it is needed, and why it is needed). With additional data sources (explained later in Section 4.1.2), we may view our problem not only as multivariate time-series, but as a system better explained by some generalized linear model. This approach naturally leads us to state-space models where an underlying, perhaps unknown process is responsible for generating our observed data. State-space models are also able to adapt even quicker than exponential smoothing methods to potential shocks to the system [17]. This responsiveness has made state-space models popular for forecasting systems subject to sudden changes, such as the stock market [17, 28], biological systems [29], or ecological systems [30]. State space models can also handle missing or potentially inaccurate data [19, 30] so they should be considered as possible suitable methods for our problem.

Hidden Markov Models (HMMs) are a type of State-Space model that can be used to describe an underlying system that changes without our observation and according to some stochastic process. HMMs initially gained popularity in describing the underlying system such as in handwriting or speech recognition [31] but have been adapted to successfully forecast time series observations like economic data [32, 33, 34, 35] and natural systems like bovine fertility [36] or volcanoes [37]. Their wide applicability and adaptability make them a suitable candidate for our forecasting problem, but HMMs may be limited by their assumption that the underlying process is a finite Markov process. It may also be necessary to consider models that relax this constraint such as those that use higher order Markov processes or semi-Markov processes [36].

2.4 Bootstrap Aggregation

Bootstrap aggregation, or bagging, may be used to improve prediction accuracy and reduce variance for classification problems [38]. A model (often a simple or even weak classifier) is fit to several bootstrapped samples of training data. The resultant predictions are averaged to form a bagged prediction that is often more accurate than the prediction produced by the same model on just the empirical training data. Bootstrapping was adopted for time-series in several different ways under several different names, but the two most popular are block and sieve. Block bootstrapping involves splitting a series into blocks of time (that may overlap) and sampling them with replacement to form new series [39]. The most relevant method to our problem is sieve bootstrapping [40]. A training series is decomposed into trend, seasonal, and residual components. The residual series is sampled with replacement and added back to the trend and seasonal components to produce a surrogate training series with approximately the same distribution of residuals. A forecast model then uses several of these surrogate series to produce several forecasts. We take the average of these forecasts. This process has been shown to be particularly helpful in series governed by comparatively large residuals and residuals that are not distributed normally [41].

3 Problem and Objectives

3.1 Problem Description

> "During a conflict, resupply quantities must constantly be reviewed and adjusted based on historical usage data gathered as the conflict progresses... A review of U.S. Army involvement in recent operations clearly indicates the need to improve logistical planning. Plans must be developed to support all levels of combat operations/ stability and support operations. It is critical that Class V (ammunition) support planning be detailed..." (FM 4-30.1: Munitions Distribution in the Theater of Operations, ch 4)

The math required to forecast food consumption on a FOB in Afghanistan is relatively straightforward. We determine how many mouths there are to feed, how many days they need food, and we multiply by three meals per day. Ammunition consumption is not only a much bigger problem;[3] it is much more complex. We don't know why ammunition will be used on any given day in the future and we don't know how much it will take to get the mission done. We don't even know how much ammunition any unit has currently available in stocks. Our best forecast must therefore be based on historic data that details the exchange of ammunition on the battlefield.

There are two different primary questions to answer in this problem. The tactical level question is how much ammunition, by type, will be needed at a location by its units during an upcoming time period (How many 5.56mm rifle rounds will the units working out of FOB Sharana need next month)? The strategic level question is how much ammunition (by class, weight, and volume) will we be needed in a theater of operations for a large time horizon (How many air or water shipments of small arms ammunition will we need in Afghanistan in 2015)? The tactical level estimate will drive decisions about ammunition allocation and distribution. The strategic level estimate will provide guidelines to the ammunition producers, sellers or buyers, and transporters. Both estimates may rely on different models and perhaps different data sources to increase their accuracy. There may also be a need for intermediate estimates. For example, the battalion supply sergeant at a FOB needs to know how much ammunition his battalion needs in the coming months, the brigade supply team needs to know for the entire FOB, the division supply team needs to know how much ammunition is needed at 20 different FOBs and needs to know with enough time to make a distribution plan, etc.

[3] Ammunition accounted for over 73% of daily sustainment requirements by weight during Operation Desert Storm [2].

So forecasts are needed at several different echelons and may be needed for different time horizons (See Figure 2).

Similarly, our problem may be organized by a hierarchy of ammunition types. At the lowest level we have individual ammunition calibers. They may be aggregated by a hierarchy of types (See Figure 3). In this research we will consider this hierarchical approach because our data set is appropriately tagged with this information but not with the unit hierarchy information. We will consider a two-level hierarchy with three types of machine gun ammunition (.50 caliber, 5.56mm, and 7.62mm) at the lowest level and the sum of all machine gun ammunition at the highest level [42].

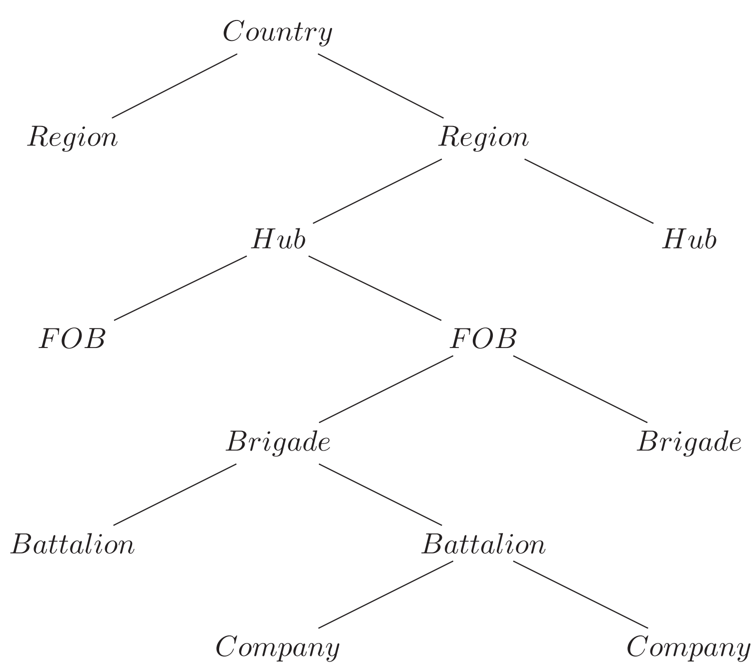

Figure 2: Hierarchy of Army Units by Space

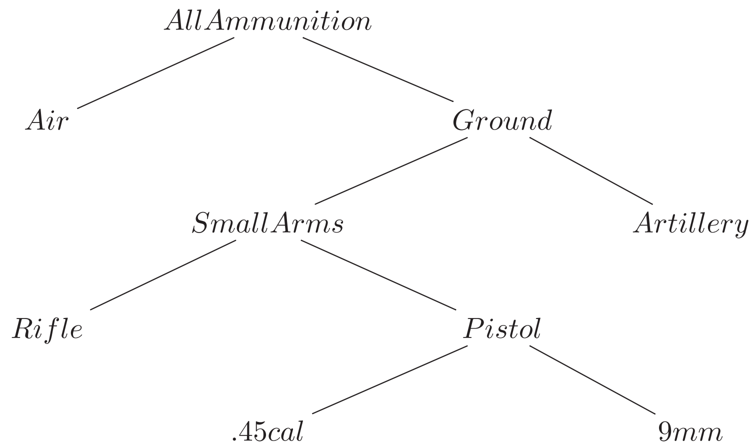

Figure 3: Ammunition Organized by Type

3.2 Problem Statement

We formally define the forecasting problem at the tactical level to estimate $N_i(m,t)$, the quantity of ammunition type i required by military unit m during time period t. At any particular FOB, f, in the set of all FOBS, F, where the set of all the units at FOB f are denoted by m_f, the quantity of interest is:

$$N_{ift} = N_i(m_f,t) = \sum_{m \in f} N_i(m,t)$$

The strategic level problem is similar in structure, but is only relevant when it includes all FOBs, F:

$$N_{it} = N_i(m_F,t) = \sum_{f \in F} \sum_{m \in f} N_i(m,t) = \sum_{m \in F} N_i(m,t)$$

Similarly, we particularly seek to forecast for a group of ammunition types, say all machine gun ammunition G, which is a subset of all ammunition I:

$$N_{Gt} = \sum_{i \in G} N_i(m_F,t)$$

3.3 Objectives

The ideal solution to our problem will provide users at all levels of war with relevant, accurate forecasts of ammunition use. Strategic planners will be able to forecast ammunition requirements over a long time horizon for a large aggregation of units. Tactical decision makers will seek shorter term, more accurate forecasts for just the units for which they are responsible. In order to achieve this end state, we must properly organize the available data and then determine which methods (of those mentioned above) make the best estimates for the different forecasting problems.

Given the current set of available data, much of which has already been aggregated at different levels or time periods, and the different echelons at which an estimate is desired (strategic or tactical), there is clearly a need to organize our data. This organization must take place prior to any forecasting in order to ensure we use the most relevant data for each estimate.

Of the possible hierarchical schemes we use to build predictive models, some will perform better than others under certain conditions. This research identifies which models are better at making a tactical level estimate and which ones are better for strategic plans. Some models will work better than others for different aggregations of ammunition type and for different forecast horizons.

4 Data and Methodology

4.1 Data

4.1.1 Ammunition Transaction Data

We have 39 months of ammunition transaction records from US Army units in Afghanistan. From January 2010 through March 2013 there were over 800,000 ammunition transactions recorded in the form: "On date D, supply unit W issued X rounds of ammunition of type Y to unit Z." The data came from the Army's Conventional Ammunition Packaging and Unit Load Data Index (CAPULDI) Standard Army Ammunition System (SAAS)[4][43].

When considering a particular unit or ammunition type, this data set is sparse. For example, of the 125 units operating around Jalalabad, 41 units made only one or two ammunition draws over a one year period. This sparseness indicates the intermittent nature of the time series and illustrates

[4] Here forth referred to simply as "SAAS data."

the difference between ammunition consumption and ammunition transactions. Our data tells us when a unit received more ammunition; we are still unclear about how much ammunition the unit has on hand or has used since the last transaction. These shortfalls prevent us from using the SAAS data to answer the question: Why is ammunition being used? This question is at the heart of our quest to predict how much ammunition will be required in the future. In order to gain a clearer picture we may need other data.

The SAAS data may still be used to forecast future ammunition demand by making time series of previous demands. In order to remove the intermittent nature of the series we aggregated the demand at country level and looked only at ammunition with a large, regular demand: machine gun ammunition. Machine gun ammunition is a perfect candidate for time series analysis because it it is drawn relatively frequently compared to other ammunition types; it is drawn by a wide variety of units (as opposed to artillery which is drawn almost exclusively by artillery units); and it is used in a wide variety of conflict types. It is intuitive that machine gun ammunition should most closely indicate the actual tempo of battle for these reasons.

Our SAAS data set is missing transactions records for April, May, and June of 2011. In order to examine the seasonality of ammunition demand we needed a series of at least two years length, so it was necessary to fill in these three months with appropriate estimates of demand. We used linear interpolation at both weekly and monthly levels to estimate the values assuming a yearly seasonal period. Simply put, we used an average of April 2010 and April 2012 values to estimate April 2011 values. This allowed us to maintain a realistic variance in the data. We confirmed this variance by examining the autocorrelation of the squared residuals; they indicated no heteroskedasticity conditional on time attributable to our interpolations.

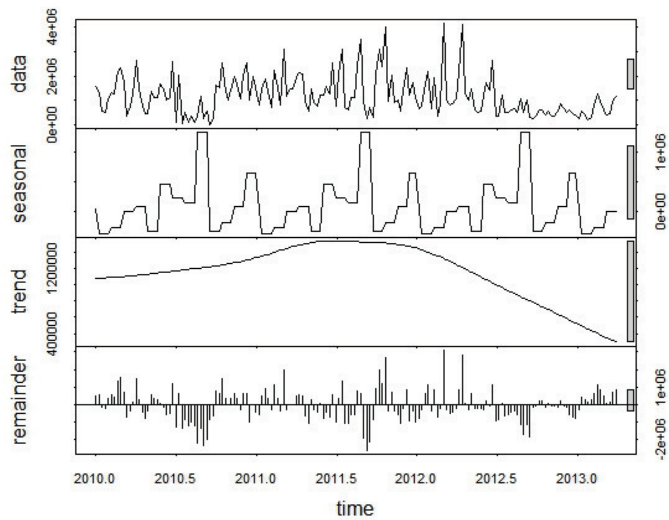

Figure 4: Decomposed time series of .50 caliber ammunition demand.

Next we attempted to organize the SAAS data by unit type and size. Of the approximately 76,000 machine gun ammunition transactions, we were able to identify the size and type of the receiving unit on about 15,000 transactions, or nearly 20%. This was enough to establish mean demand amounts per generic unit types, but not enough information to conduct time series analysis on the data by unit type.

The time series of each ammunition type are non-stationary processes with trend and seasonal components. The seasonality of the series are different for different ammunition types and are sometimes negligible. The residuals of each series are very close to normally distributed and show little or no autocorrelation; they can therefore be modeled as stationary residuals for our forecast methods. Augmented Dickey-Fuller tests on each series support this by significantly rejecting the hypothesis that any series is characterized by a unit root. See Figure 4 for the decomposed series of .50 caliber ammunition demand. Note the large range of the residual series (5.7 million) compared to the trend (1.2 million), seasonal component (1.7 million), and the total series (4.1 million). This comparatively large residual series is common to all the series in this research and helps explain the relatively high errors achieved by even our best forecasts.

4.1.2 Auxiliary Data

Other helpful historical data includes how many troops were in Afghanistan at any given time, how units were organized (who worked for whom, and unit makeup), and what significant activities (SIGACTS) occurred on the battlefield. We can use each of these data sources to help describe the workings of the entire system at each snapshot in time. They provide additional attributes to each ammunition transaction instance and potentially allow for more accurate predictions. Afghanistan troop levels [44], including US Forces, Afghan Security Forces, and other Foreign troops, provide a country-wide indicator of total monthly activity. Unit task organization (TASKORG) is available through "Order of Battle" documents [45] that describe which units worked together and where they worked on the battlefield. This monthly data can be used to hierarchically organize ammunition transactions by unit location, and serves as the input data necessary to make forecasts using OPLOG Planner. SIGACT data [46], even though it has been aggregated over space[5] and time, is still capable of helping to describe the causes of ammunition consumption and the effects of plans and battlefield conditions. This data can provide the link between battlefield conditions (like time of year and troop levels) to actual ammunition consumption. While it will be difficult to accurately predict future SIGACTs, the relationships we learn about conditions, SIGACTs, and ammunition transactions can be expected to continue to future time periods.

4.2 Methodology

4.2.1 Data Organization

The data lend themselves to aggregation by ammunition type, echelon of Army unit, or even time. By adding attributes to data entries that reflect their place in a hierarchy instead of merely summing their values, we can better describe the characteristics of the data at different levels. This allows us to create separate data matrices for each hierarchy level (as advocated by [47]) and still affords us the opportunity to test other organization schemes without creating a new data set. It also allows us to later create models with three different hierarchical schemes: disaggregated models that

[5] Aggregation over space, for example, by province in Afghanistan, is akin to unit aggregation. Military units are often responsible for a designated space and subordinate units have smaller divisions of the space. It is therefore acceptable to equate the aggregation of units (several battalions lumped into one Brigade Task Force) with the aggregation of space (several districts lumped into one province). This concept will appear throughout the methodology portion of this proposal.

forecast based on the individual attributes, a pooled model that forecasts based on a summed total of the attributes, and an intermediate model which uses the all the labeled individual attributes at once.[6] It will then be necessary to experiment with different hierarchical organization schemes. For example, we may split our attributes into four levels: unit, FOB, Province, and Country (See Figure 2 for another possible (more complete) breakdown of the unit-based hierarchy). We will build forecast models with these different schemes and compare their results. Organization by space/unit will be akin to a producer of one product estimating demand over several geographic or demographic markets. We could also base the hierarchy on ammunition types (See Figure 3) or on a combination of unit and ammunition types. Organization by ammunition type will be more similar to the way production companies estimate sales for several products that can be organized by type, brand, etc.

Due to the sparse nature of the SAAS data, it is necessary to aggregate the data by time. As mentioned previously, many units drew ammunition only once or twice a year. Using a day as our time unit results in a data set that is too intermittent to forecast accurately. We may aggregate transactions at the week or month level to provide a more full data set that can now be used to predict in units of weeks or months ahead (as opposed to days). We experiment with different time based aggregations of the data in order to determine the scheme that provides the most accurate forecasts.

4.2.2 Forecasting

Once the data are properly organized and we examine the trends, variances, and correlations of the variables, we are poised to make forecasts. For each forecast task, we consider three main methods and three control methods. ARIMA models, Exponential Smoothing models, and Hidden Markov Models are compared to naive forecast methods.

4.2.2.1 Naive Methods In order to properly determine the effectiveness of the below forecasting methods, we compare their results to simple methods currently in use. First, we form a naive forecast that assumes next time period will incur the same ammunition demand as the time period

[6]For example, machine gun ammunition is composed of three different calibers. If we seek to forecast the total amount of machine gun ammunition required, we would make three separate dissagregated models and sum their results(one for each caliber), one pooled model will take as inputs the sums of each of the caliber amounts, and the intermediate model will take three inputs per time period- one for each caliber. The disaggregated and pooled models ignore the inherent covariance structure of the three calibers while the intermediate model uses this information to potentially increase forecast accuracy.

that precedes it. Extensions to this method include a yearly naive forecast that assumes next time period will incur the same ammunition demand as the same time period last year, and a naive forecast that also includes random drift.

We also use the OPLOG Planner to produce an estimate based on the units expected to be in theater and their assigned missions. Using a similar construct as OPLOG planner, we make estimates based on the average usage rates from the current data set; these are referred to as generic unit estimates.

4.2.2.2 Time Series The exponential smoothing approach provides a forecast at any level in our unit or ammunition hierarchy with the following three equations:

$a_t = \alpha(N_{ift} - s_{t-p}) + (1 - \alpha)(a_{t-1} + b_{t-1})$

$b_t = \beta(a_t - a_{t-1}) + (1 - \beta)b_{t-1}$

$s_t = \gamma(N_{ift} - a_t) + (1 - \gamma)s_{t-p}$

where a_t, b_t, and s_t are the estimated mean, slope, and seasonal effect at time t, respectively, and α, β, and γ are our smoothing parameters. We make forecasts of ammunition consumption for each type of ammunition over several different time horizons. We make aggregated forecasts at applicable echelons based on ammunition type or unit location. We then investigate how the accuracy and robustness of our forecasts change as we adjust the amount of past data used as training inputs (and as we adjust the α parameter). This helps determine the "freshness" required of our data in order to produce an acceptable forecast. We also use a multivariate approach of the same method.

The seasonal ARIMA process is given by:

$\Phi(B^m)\phi(B)(1 - B^m)^D(1 - B)^d y_t = c + \Theta(B^m)\theta(B)\epsilon_t$

where $\{\epsilon_t\}$ is a white noise process with zero mean and σ^2 variance, B is the backshift operator, $\phi()$ and $\theta()$ are polynomials of order p (autoregressive order) and q (moving average order), respectively. $\Phi()$ and $\Theta()$ are polynomials of order P (seasonal autoregressive order) and Q (seasonal moving average order), respectively. d and D are the non-seasonal and seasonal difference orders, respectively. m is the seasonal frequency. We select the order terms by examining the AIC of models fit under different orders and choosing the parameters of the model with the lowest AIC[18].

4.2.2.3 State-Space and Hidden Markov Models The above exponential smoothing framework may not adapt quickly enough to some of the societal changes we expect to be important in our data. A natural next approach would be a generalized linear model with coefficients that change over time. A multivariate autoregressive state space model can approximate such an approach where the process (as depicted in Figure 5) is modeled by:

$\mathbf{X}_t = \mathbf{B}\mathbf{X}_{t-1} + \mathbf{w}_t$

Where $\mathbf{X}_t$ is a vector of state values at time t, $\mathbf{B}$ is a transition matrix that describes how states change over time, and $\mathbf{w}_t$ is a multivariate error term that is distributed normally with mean zero and variance covariance $\mathbf{Q}$. The observations of the model are modeled by:

$\mathbf{N}_t = \mathbf{Z}\mathbf{X}_{t-1} + \mathbf{v}_t$

Where $\mathbf{N}_t$ is a vector of observations, $\mathbf{Z}$ is an emission matrix that describes how states produce observations, and $\mathbf{v}_t$ is a multivariate error term that is distributed normally with mean zero and variance covariance $\mathbf{R}$.

The two above equations form the base of a recursive process in which we smooth, filter, predict, and forecast in order to estimate future observations. Smoothing gives us the best estimate of past states given past observations. Filtering makes the best estimate of the current state given past and current observations, prediction estimates future values of the state, and we forecast to estimate future observations based on future state values.

From the above state-space approach, if we then assume that the underlying system can take only a finite number of states [7] and transitions to those states can be assumed stochastic and memoryless, we can then use a hidden Markov model to describe the system and make inferences about the observed outcomes.[8] Under this approach, we seek to identify the state at a particular time period (X_t) in order to estimate the quantity of ammunition that will change hands in that time period (N_{it}), as depicted in Figure 6).

[7] For example, each ammunition consuming unit may take only two states: the unit needs more ammunition, or it does not. We will experiment with several different numbers of states in order to find a model that works best for each level of the system- maybe a two state model at the battalion level and a model with more states at the Country level.

[8] The auxiliary data sources such as number of troops present in country or number of monthly insurgent-initiated attacks may seem like inputs to the system while the actual ammunition transactions are outputs. It is a trivial change to our thinking about this system that makes all data sources outputs of the underlying system (war); so we will model all these data as a multivariate set of observed outcomes [48]. The rest of this section will display only the univariate case for clarity.

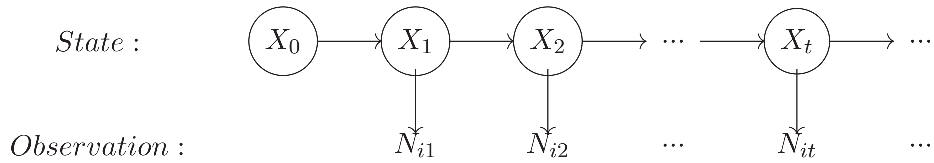

Figure 5: State Space Model Diagram

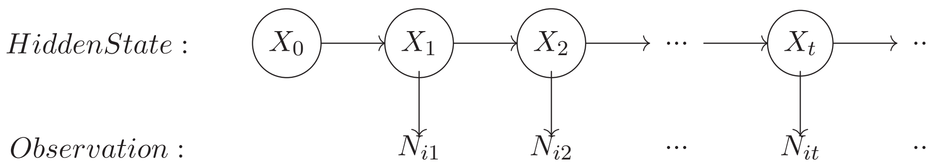

Figure 6: Hidden Markov Model Diagram

The parameters of the model include the number of possible states (S), the transition probabilities to and from these states (A), the probabilities of observations given the current state (B), and the initial probability distribution of states (π). We will denote all these parameters by λ. After selecting initial model parameters, we seek to identify the most likely sequence of states given the observations up to time t: $P(X_0 : X_t|\lambda, N_{i1}...N_{it})$. Then we will use a variation of the EM algorithm to recursively improve our estimates of the state sequence and the model parameters in order to better match the sequence of observations. After finding the best model parameters (λ), and most likely state sequence ($X_0 : X_t$), we can then estimate the next state (X_{t+1}), and forecast the next observation (N_{it+1}), or set of observations in the multivariate case, by maximizing $P(N_{it+1}|\lambda, X_0 : X_{t+1})$. In a multivariate model where we have observations for some variables and not for others in a future time period, [9] we can use the same model framework to make a prediction using this additional information by maximizing

$$P(N_{it+1}|\lambda, X_0 : X_{t+1}, P_{t+1})$$

[9] For example, we know how many troops will be in Afghanistan next month but don't know how much ammunition they will consume.

where P_{t+1} represents a vector of the observations of auxiliary variables at time $t+1$ and the model parameters (λ), and the previous state sequence ($X_0 : X_t$), were estimated using the previously mentioned recursive algorithm on multivariate time series data as opposed to just the univariate data.

4.2.2.4 Forecast Improvement From the above methods, we choose the best performers as candidates for potential improvement by bootstrap aggregation, or bagging. Here we make a number of similar forecasts from different time series created by sampling the residuals of the original series. We take the average of these forecasts. The prediction intervals of the forecasts may also be averaged as long as they are calculated similarly. All of our models calculate the prediction intervals assuming a normal distribution of error, so they may be averaged just as the point forecasts are.

4.2.3 Evaluation

Forecast models are compared to one another based on their respective mean absolute percentage errors (MAPE) on several different forecasting tasks and training data sets. First, we use monthly series, then weekly. Models are tested with just univariate data (only the SAAS data), as well as with multivariate "input" data such as US troops levels and the number of insurgent initiated attacks. Models that perform well with fewer inputs will be preferred over those with more inputs because data dissemination is a difficult and often error-inducing task on the battlefield. The SAAS data may be disaggregated by ammunition type (the number of total machine gun rounds can be split into the number of 5.56mm rounds, 7.62mm rounds, and .50 caliber rounds).

Methods are tested with pooled data (all machine gun rounds in all of Afghanistan) and disaggregated data (we will consider the sum of forecasts for each caliber of machine gun round). Models are also be tested at different forecast horizons: from one to 12 months ahead and from one to 12 weeks ahead. After all this model comparison, we are able to assign the best model (of those tested) for any particular scenario. Our comprehensive forecasting engine will choose the correct model based on the parameters of the question. For example, when decision makers need a forecast for a small unit (battalion) over a small time horizon (one month), the engine uses one particular model to forecast; when another decision maker needs a forecast for a larger unit (division) over a longer time horizon (six months), the engine uses a different, more accurate model to generate the forecast.

Initial forecast results were high in error because predicting the amount of ammunition demanded in a particular week is a tough task. We considered that this task may be less relevant that predicting the aggregate demand over two weeks. For example we may still make weekly forecasts, but we calculate our error by comparing the sum of our one and two-week ahead forecasts with the sum of the actual demand for one and two weeks ahead. This should allow for military planners to see more relevant error statistics and aid in their decisions for model selection.

5 Analysis

5.1 Data Analysis

After an initial cleaning and organizing, we determined several things about our data that helped shape our analysis. First off, the individual time series are intermittent or lumpy. Just as in most combat related data sets, [10] ammunition transactions are few and far between when viewed at the unit level. They are often large as well. It is interesting to note that the smaller transactions (one type of ammunition, say, as opposed to 10 different types) may very well be the most descriptive and the closest in time to actual needs, whereas the larger transactions are presumably more routine. This, combined with the lack of unit information in the data set, confirms our decision to aggregate data over several units. Individual ammunition types exhibit normal distributions of issued quantities and their aggregates look like the sum of these normal distributions. The residual and error terms associated with our models also exhibit normal distributions. By decomposing the SAAS data time series we revealed a high correlation (.73) between the trend component and the US troop levels time series. This is intuitive but encouraging because the troop level data may prove a useful series for the multivariate methods.

[10] War is often described as hours and days and weeks of boredom interrupted by seconds and minutes of unbridled chaos.

5.2 Forecasts

For an initial test of the forecasting methods, we made a monthly time series of transactions of machine gun (MG) ammunition aggregated at the country level. We chose machine gun ammunition (composed of three different calibers and 12 different individual types) because it was drawn by virtually all units throughout the 39 month period, and it was drawn relatively frequently. This frequency is a result of the wide application of machine guns in combat- their use is applicable to a wide array of conflicts and so is closely related to the overall combat intensity in an area or over a time period. Multivariate models also used monthly counts of US troop levels in Afghanistan and monthly counts of insurgent initiated attacks. Forecasts were made at various time horizons from one month to one year in the future. The results of these initial forecasts are depicted in Table 2 and the individual models and their results are explained further in Appendix 1. The Naive method (which assumes next month's demand will equal the previous month) and exponential smoothing performed the best at forecasts made one or three months out. A three-state univariate hidden Markov model performed the best at six months out and beyond. These initial results demonstrate the necessity of the methodology we have outlined that tests each method with several different types of data and model parameters.

Method: Forecast Horizon	Naïve Monthly	Exponential Smoothing	3 State HMM
1 Year	205.6% +/- 99.2%	231.1% +/- 107.5%	54.7% +/- 64.4%
9 months	172.5% +/- 126.5%	203.4% +/- 210.2%	41.6% +/- 35.0%
6 months	86.3% +/- 70.7%	94.6% +/- 78.8%	34.0% +/- 21.1%
3 months	42.6% +/- 33.1%	42.7% +/- 34.0%	43.5% +/- 19.2%
1 month	32.0% +/- 15.4%	31.9% +/- 16.3%	42.5% +/- 17.5%

Table 2: Mean Absolute Percentage Error for the best monthly forecasting models. Models made monthly forecasts for the quantity of machine gun ammunition demanded in the entire country of Afghanistan. Lower percentages indicate lower error. Errors over 100% indicate that a model tended to overestimate by at least a factor of two.

Next we created weekly time series of the four machine gun ammunition types (.50 caliber, 5.56mm, 7.62mm, and all three combined). The Naive models again performed fairly well. The multivariate models always performed worse than their univariate counterparts and will no longer be considered in this research. Their poor performance is most likely due to the monthly aggregation of the auxiliary data sets. Although it makes sense that the other data sources, such as troop levels and attack counts, should only aid a model in forecasting ammunition demand, all the multivariate models suffered when more information was added. This also may have to do with the non-intuitive orders of demand and attacks. Sometimes high levels of enemy activity are preceded by large ammunition demand, sometimes they are followed by it. The remaining contending models are the univariate HMM (with 3 states), ARIMA, and exponential smoothing methods. They performed reasonably well and the HMMs were sometimes able to beat the naive forecasts. When we bagged the ARIMA and ES models, however, their performance improved, but generally not enough to beat the naive method.[11] Bagging was performed both with and without replacement of the residuals. The bagging without replacement tended to perform better, but not significantly so. We experimented with Croston's forecasting method and found it unsuitable for our data due to the level at which we aggregated the series.[12] We will therefore no longer consider any intermittent forecasting strategies for our research. See Table 3 for a summary of results on the actual .50 caliber data.

Weeks Ahead:		1	2	3	4	5	6	7	8	9	10	11	12
# Samples:		13	12	11	10	9	8	7	6	5	4	3	2
Exponential Smoothing (stlf)		**0.639457**	0.836136	0.860696	**0.820839**	**0.686928**	0.584935	0.687353	0.786809	0.781632	0.9345	**0.396123**	0.53399
	95% conf int	0.383136	0.660139	0.606496	0.533857	0.450449	0.416315	0.680226	0.95768	1.1918	1.702493	0.568068	0.123706
Exponential Smoothing with bagging (w/o replacement)		0.725142	**0.620009**	0.836523	1.024502	1.024891	0.571567	0.469657	**0.483196**	1.273939	**0.335081**	0.621367	**0.272605**
	95% conf int	0.700147	0.333541	0.712057	1.022416	1.32016	0.277127	0.214028	0.25508	1.603918	0.518982	0.186928	3.321232
Exponential Smoothing with bagging (w/ replacement)		0.772723	0.614781	**0.664226**	1.339147	0.692075	**0.442029**	**0.367042**	0.574464	**0.348427**	1.110568	2.141963	0.347333
	95% conf int	0.413208	0.458897	0.619328	1.441963	0.693568	0.212123	0.214894	0.493709	0.247945	1.39194	6.668023	4.329643
ARIMA (stlf)		0.629236	0.841719	0.877159	0.808622	**0.67702**	0.584024	0.684072	0.769979	0.777669	0.915794	**0.403021**	0.540327
	95% conf int	0.38435	0.666529	0.617357	0.517635	0.443348	0.410088	0.680927	0.905153	1.187448	1.637278	0.54003	0.065065
ARIMA with bagging (w/o replacement)		0.756778	**0.713132**	0.667726	1.023127	0.771559	**0.405751**	0.414156	**0.34116**	**0.564317**	0.987193	1.126521	0.507137
	95% conf int	0.872719	0.366951	0.301053	0.868173	0.440411	0.170888	0.244858	0.196057	0.346793	1.420982	2.098652	4.674641
ARIMA with bagging (w/ replacement)		**0.610119**	0.820662	**0.565171**	**0.730844**	1.397788	0.435476	**0.363365**	0.727776	0.666218	**0.881639**	0.721915	**0.462213**
	95% conf int	0.387891	0.642775	0.175443	0.582251	1.487688	0.185164	0.194403	0.726406	0.396442	0.469719	1.671797	0.832662
3-state HMM		**0.460192**	**0.47595**	**0.453016**	**0.448809**	**0.403465**	**0.457111**	**0.48123**	**0.417331**	**0.380446**	**0.380723**	0.485356	0.650479
	95% conf int	0.143408	0.151565	0.154102	0.195512	0.174356	0.191526	0.226279	0.270445	0.335285	0.495158	0.709054	0.376197
3-state HMM with bagging (w/o replacement)		0.703262	0.657907	0.724761	0.823366	0.596412	0.50672	0.522696	0.697808	0.507006	0.511367	**0.40555**	0.253435
	95% conf int	0.295505	0.279441	0.240422	0.501877	0.194795	0.151518	0.345832	0.247933	0.42006	0.339991	0.761179	0.76841
3-state HMM with bagging (w/ replacement)		0.547535	0.670369	0.662839	0.842026	0.767014	0.765391	0.68603	0.493468	0.54326	0.680495	0.591973	**0.205605**
	95% conf int	0.195843	0.356211	0.228696	0.471622	0.381832	0.221401	0.240158	0.429168	0.444576	0.328804	0.799542	1.459408
Naïve (without drift)		**0.47548**	0.706135	0.693135	**0.687647**	**0.479874**	0.399303	**0.448454**	**0.524381**	0.480779	**0.400494**	0.472026	0.699091
	95% conf int	0.156243	0.306131	0.378466	0.369106	0.176474	0.187901	0.277373	0.285746	0.29657	0.413308	0.805652	1.228403
Naïve (without drift) with bagging (w/o replacement)		0.801686	**0.595015**	**0.52003**	0.964628	0.592713	0.440091	0.486147	0.664833	**0.471497**	0.805718	0.678096	**0.438886**
	95% conf int	0.727076	0.155549	0.199088	1.021626	0.209567	0.215622	0.15215	0.387883	0.685832	0.569541	0.468397	2.73584
Naïve (without drift) with bagging (w/ replacement)		0.704579	0.66034	0.598248	1.615241	0.944388	0.459658	0.84069	0.52928	1.325158	1.119005	**0.40362**	0.646086
	95% conf int	0.15583	0.488386	0.186648	2.381428	1.123584	0.265757	0.319904	0.322518	1.332791	1.520482	0.703705	4.398195
Naïve (with drift)		**0.47586**	0.708326	**0.687206**	**0.6731**	**0.475716**	**0.437291**	0.493552	0.568141	**0.52806**	**0.456513**	**0.527106**	0.7457
	95% conf int	0.151647	0.302006	0.36862	0.344273	0.177864	0.198641	0.282811	0.284294	0.309822	0.333882	0.642056	0.911881
Naïve (with drift) with bagging (w/o replacement)		0.510957	**0.666899**	2.766584	1.197534	0.587349	0.478364	0.567679	**0.386152**	0.531491	0.515298	1.261366	**0.40546**
	95% conf int	0.233215	0.407369	2.767005	1.456362	0.216698	0.152094	0.183988	0.330824	0.362049	0.500238	2.133342	2.060531
Naïve (with drift) with bagging (w/ replacement)		**1.257048**	0.696981	0.977004	0.809814	0.573307	0.750736	**0.420354**	0.575114	1.399518	1.007324	0.575352	0.522236
	95% conf int	1.109039	0.597495	0.873008	0.383706	0.206232	0.187729	0.171091	0.337401	1.410683	2.377389	0.96285	1.94036

Table 3: Mean Absolute Percentage Error and 95% confidence intervals for forecasting models on the .50 caliber time series. Models made weekly forecasts for the quantity of ammunition demanded in the entire country of Afghanistan. Lower percentages indicate lower error.

See Table 4 for the improvements gained by bagging the time series forecast methods on all series. Bagging tended to improve forecasts, but not in a statistically significant manner. We then

[11] We also experimented with bagging the naive and hidden Markov models- bagging only degraded their performance.

[12] Most of our series are composed of intermittent series, which, when aggregated, form non-intermittent series.

	Weeks Ahead:	1	2	3	4	5	6	7	8	9	10	11	12
	# Samples:	13	12	11	10	9	8	7	6	5	4	3	2
Fifty Caliber	Exponential Smoothing	**0.639457**	0.836136	0.860696	**0.820839**	**0.686928**	0.584935	0.687353	0.786809	**0.781632**	0.9345	**0.396123**	0.53399
	95% conf int	0.383136	0.660139	0.606496	0.533857	0.450449	0.416315	0.680226	0.95768	1.1918	1.702493	0.568068	0.123706
	ES-Bag	0.725142	**0.620009**	**0.836523**	1.024502	1.024891	**0.571567**	**0.469657**	**0.483196**	1.273939	**0.335081**	0.621367	**0.272605**
	95% conf int	0.700147	0.333541	0.712057	1.022416	1.32016	0.277127	0.214028	0.25508	1.603918	0.518982	0.186928	3.321232
	ARIMA	**0.629236**	0.841719	0.877159	**0.808622**	**0.67702**	0.584024	0.684072	0.769979	0.777669	**0.915794**	**0.403021**	0.540327
	95% conf int	0.38435	0.666529	0.617357	0.517635	0.443348	0.410088	0.680927	0.905153	1.187448	1.637278	0.54003	0.065065
	ARIMA - Bag	0.756778	**0.713132**	**0.667726**	1.023127	0.771559	**0.405751**	**0.414156**	**0.34116**	**0.564317**	0.987193	1.126521	**0.507137**
	95% conf int	0.872719	0.366951	0.301053	0.868173	0.440411	0.170888	0.244858	0.196057	0.346793	1.420982	2.098652	4.674641
5.56 mm	Exponential Smoothing	**0.593966**	0.87752	1.021841	1.043767	**1.003157**	0.987387	1.322139	1.292363	1.428464	1.235478	**1.186909**	**0.375207**
	95% conf int	0.344995	0.313801	0.389491	0.497513	0.51913	0.62419	0.836077	0.845928	1.44172	1.686594	3.277718	1.707554
	ES-Bag	1.164003	**0.70167**	**0.891454**	**0.60139**	1.467874	**0.306933**	**1.273274**	**0.923663**	**0.353891**	**0.462412**	2.16761	0.639573
	95% conf int	0.848779	0.343439	0.553388	0.307527	1.252399	0.235684	1.844218	1.1317	0.354256	0.725481	7.261791	3.001756
	ARIMA	**0.86433**	1.499105	1.873161	2.060831	2.130119	2.307447	2.616282	2.524654	2.557935	2.164977	1.808727	0.710693
	95% conf int	0.510566	0.720321	0.814978	1.060563	1.17315	1.468851	1.578454	1.6867	2.245979	2.874062	5.048095	8.681016
	ARIMA - Bag	0.942595	**0.932277**	**1.225074**	**1.349668**	**0.429189**	**0.451718**	**0.803105**	**0.838792**	**0.874082**	**1.204496**	**0.957875**	**0.612833**
	95% conf int	0.913079	0.887562	1.298509	1.777788	0.281734	0.125135	1.115969	1.194437	0.915161	2.058303	2.649203	7.078858
7.62 mm	Exponential Smoothing	0.699597	**0.977326**	0.992768	1.386371	1.374133	1.398858	1.503129	0.758933	0.741653	0.602504	**0.265107**	**0.137611**
	95% conf int	0.73752	1.180132	1.153506	1.773898	1.825313	1.573912	1.753783	0.653734	0.567736	0.591819	0.162433	1.211227
	ES-Bag	**0.659331**	1.004022	**0.569266**	**0.534946**	**0.479987**	**0.948958**	**0.605519**	**0.225281**	**0.36738**	**0.224343**	0.333079	0.694822
	95% conf int	0.582522	0.914266	0.570051	0.501425	0.525594	1.542425	0.688048	0.297831	0.348121	0.071719	0.806093	4.016814
	ARIMA	**0.772959**	1.13513	1.193554	1.495723	1.501642	1.675348	1.833252	1.020239	1.017331	0.909199	0.495029	**0.466694**
	95% conf int	0.756538	1.258249	1.211437	1.623925	1.814964	1.83163	2.086026	0.818625	0.906285	1.176266	0.546359	3.160184
	ARIMA - Bag	1.28715	**0.80372**	**0.894021**	**1.427691**	**0.711614**	**0.697857**	**0.421117**	**0.202097**	**0.856802**	**0.339039**	**0.405793**	1.607891
	95% conf int	1.067856	0.689839	0.598354	1.583027	0.778035	0.717386	0.300971	0.163865	1.035511	0.417812	0.960576	11.82518
All MG Ammo	Exponential Smoothing	**0.449336**	0.59231	**0.575737**	0.59454	0.439057	0.441382	0.550727	0.624562	0.664627	**0.540026**	**0.222603**	**0.176869**
	95% conf int	0.253477	0.413695	0.383748	0.438285	0.275825	0.281036	0.526519	0.649275	0.644198	1.124352	0.46624	0.486275
	ES-Bag	0.529777	**0.448029**	0.65152	**0.393562**	**0.296056**	**0.349236**	**0.337368**	**0.342361**	**0.465747**	0.548615	0.498332	0.547022
	95% conf int	0.277914	0.171731	0.565746	0.170581	0.113324	0.131067	0.312428	0.223804	0.322868	0.916052	0.384525	2.745503
	ARIMA	**0.629236**	0.841719	0.877159	**0.808622**	**0.67702**	0.584024	0.684072	0.769979	0.777669	0.915794	**0.403021**	0.540327
	95% conf int	0.38435	0.666529	0.617357	0.517635	0.443348	0.410088	0.680927	0.905153	1.187448	1.637278	0.54003	0.065065
	ARIMA - Bag	0.714399	**0.723842**	**0.849585**	0.933377	0.712421	**0.306076**	**0.330217**	**0.368365**	**0.368924**	**0.411535**	0.754206	**0.294972**
	95% conf int	0.34482	0.664224	0.829429	1.442894	0.62812	0.208901	0.2004	0.117949	0.24888	0.062497	2.118842	0.219424

Table 4: Mean Absolute Percentage Error and 95% confidence intervals for forecasting models on all machine gun series. Models made weekly forecasts for the quantity of machine gun ammunition demanded in the entire country of Afghanistan. Bagged models tended to improve performance over their respective base models, although not significantly so.

experimented with alternate bagging methods. We noticed of the 100 bagged forecasts, oftentimes half would be close to the actual demand and the other half would be very far off. We determined this was due to the comparative size of the residuals to the entire series. When we just sampled a fraction of the residual (one half, one tenth, or one hundredth) and added this to the original series to form a surrogate, the bagged forecast improved. It improved most with the one tenth residual method. by adding a fraction of the residual, however, we noticed that our prediction intervals were no longer accurate. This is because the bagged forecasts did not follow the original distribution of the residuals and instead followed a tighter distribution. This calls for a simple adjustment to the intervals based on the fraction used of the residual to make the surrogate series. Using a paired t-test, we compared the MAPE values for the bagged and unbagged ARIMA and exponential smoothing models. The improvement gained by the one tenth bagging procedure was statistically significant at the 95% level for both methods, over all four data sets, and for all forecast horizons up to 12 weeks in the future. There was one exception: the 2 week ahead forecasts made on the .50 caliber series by the ARIMA models were not statistically significantly improved by the bagging procedure. The remainder of results listed as bagged are one-tenth fractional residual bagging results. Although the bagging procedure is demonstrated to improve forecast accuracy in a statistically significant manner, the improvements are slight compared to the additional computational requirements. It takes 100 times longer to form a bagged forecast (one based on 100 realizations of the series with bootstrapped residuals), that only slightly increases forecast performance.

Because of the large confidence intervals shown in Tables 3 and 4, we decided to create 100 surrogate data sets for each time series by bootstrapping the residuals after a season and trend decomposition. The resulting errors tend to be greater than those actually realized on the real data, but provide much more information about how well bagging works and which methods will prove to be more robust to such seemingly random data. Table 5 shows these errors and demonstrates the forecast improvement of the one tenth bagging method for all methods and time horizons. Table 6 displays the errors over two-week periods, as mentioned in Section 4.2.3. The hidden Markov models' good performance on actual data (Table 3) and poor performance on surrogate data (Table 5) demonstrate its lack of robustness to data high in residual error. The ARIMA and ES models were more robust; see Table 7 for the typical parameters of the time series models (typical over the 6500 models made for each series: 100 surrogate series and 65 different training sets from 104 to 169 weeks long). We also considered a bottom-up tally for the combined series to see which method was more accurate for forecasting the higher level series, keeping in mind that the bottom-up forecasts generally take three times the time with the same computing power. The bottom-up forecasts did not improve over the pooled forecasts. Results are shown in Appendix 2.

Next we compared the generic unit estimates based on average values of our Afghanistan data set to the results generated by OPLOG Planner. We used the Order of Battle document[45] to provide the input data of number and type of units for both methods. Both methods tended to greatly underestimate the ammunition demand, but our method based on more recent data outperformed OPLOG planner. See Table 8 for results and Appendix 2 for complete weekly estimates. Although we did not create a suitable replacement for OPLOG planner (because of the still relatively high error), we did justify the use of a forecast method based on recent data like ARIMA or exponential smoothing over an historic average estimate when data is available. A system like OPLOG Planner still has a place where there is no recent and relevant historical data.

6 Research Contributions and Recommendations

The current methods in use proved to be effective for their simplicity and robustness. Improvements can be made to OPLOG planner and naive methods, however, by using recent time-series data to forecast demand. These forecasts can be further improved by techniques like bagging. And bagging itself may be improved, at least for series like these with comparatively large residuals, by fractional residual bagging. The bagging procedure, while a fine contribution in its own right to the overall forecasting problem, is not suggested for use by Army units on individual workstations to produce weekly forecasts because it is so computationally intensive. It may be implemented successfully, however, on a cloud-based infrastructure where more processing power may be brought to bear on the forecasting algorithms from a remote source. The bagged exponential smoothing forecasts are best for strategic level forecasts made in the near future, while naive methods are best for the tactical level forecasts. Forecasts made far into the future or for scenarios with little or no relevant data should be made using naive methods that use historical averages to estimate future demand.

We have identified the best time series methods for this particular problem and demonstrated the accuracy improvement potential of ensemble forecasts. We have also outlined a possible framework through which to use these forecasts to drive distribution and delivery decisions.

The methodology of this research will also serve as a contribution to the field of forecasting in a dynamic, hierarchical environment. By examining the different needs of decision-makers in a system, future researchers can use a method similar to this research to organize their best models by the questions they answer. This organization will offer the best answers for specific questions, rather than trying to fit a single, complicated (and perhaps uninterpretable) model that may poorly answer most questions. The result will be accessible and understandable forecasts that feed other planning systems to help leaders make better decisions about commodity demand and delivery on the battlefield in order to decrease risk and increase reliability of logistics resupply.

At the conclusion of this research, several recommendations are presented to military planners and other interested parties. First, while the OPLOG Planner proved unsuitable to provide specific, relevant ammunition demand forecasts, its methodology of historic averages should be maintained for use in a new environment where no recent and relevant data is available. It should also be updated to include ammunition transaction information from recent conflicts. Once an operation is established, the naive method will provide decent forecasts until there is enough data to use the above mentioned time-series methods- particularly exponential smoothing. Time series methods will produce the best forecasts because they exploit the autoregressive nature of demand series. Second, the Naive method is a suitable alternative when a unit lacks the capability to make more advanced forecasts. The naive method will be particularly applicable for host-nation units who need a system of estimation even if it is rudimentary. For example the Afghan National Police (ANP) and Army (ANA) are just beginning to understand the importance of logistics on the battlefield. They also lack the resources to make accurate forecasts. The naive method is easy for US advisers to explain, easy to understand, requires no sophisticated software, and works fairly well. In this case, the naive method would be the first choice for a forecast. Third, in order to make better forecasts in the future, data collection and organization systems should be modified. Unit data (size and type) should be captured more reliably. The SAAS system should prevent a single bullet from being logged more than once as being issued from the same supplier. It may be issued twice, but the second time it must be issued from the receiver of the first transaction. Currently, multiple transactions of the same bullets are recorded as multiple transactions from the same supplier- this creates confusion in the actual demand amounts over time. If the transactions were geo-referenced, another set of tools in the spatial analysis family could be brought to bear on this problem.

Finally, we have a few recommendations for future research. With additional auxiliary data (or the same data interpolated to include different weekly values), the multivariate methods may perform better than they did in this research and should be explored further. The hierarchical structure of our SAAS data was not fully exploited because of lack of information on the units receiving ammunition. With this additional information, forecast accuracy has the potential for improvement and implementation of the forecasts can be made more useful for units at various echelons in the unit hierarchy, particularly with intermediate models as mentioned previously. Also, an effort should be made to identify the on-hand amounts of ammunition at units. This information will help to better model the actual demand of ammunition use as opposed to the demand to replace stores of ammunition. Alternatively, information should be collected on actual ammunition use and then linked to events. We already have sophisticated and accurate methods to predict enemy activity. These existing methods, with the right auxiliary data, have the potential to produce accurate ammunition demand forecasts.

	Weeks Ahead:	1	2	3	4	5	6	7	8	9	10	11	12
Fifty Caliber	Naïve	0.715938	0.813259	0.841109	0.906671	0.846709	0.756485	0.774917	0.759576	0.748094	0.780319	0.855357	0.96165
	95% conf int	0.02675	0.026913	0.024943	0.02781	0.026766	0.025928	0.017858	0.0206	0.018745	0.026385	0.040555	0.045948
	ARIMA	0.704205	0.946689	1.000951	1.019513	1.030721	1.028909	1.056476	1.039649	1.079372	1.078325	1.04125	1.052707
	95% conf int	0.024626	0.028657	0.030143	0.030351	0.030887	0.031742	0.032896	0.034773	0.036785	0.035125	0.033243	0.031715
	ARIMA-Bag	0.692497	0.941552	0.994163	1.009215	1.015726	1.017346	1.047689	1.030828	1.068421	1.068255	1.032682	1.04476
	95% conf int	0.024236	0.028244	0.02956	0.029835	0.030338	0.031155	0.032217	0.033856	0.035767	0.03428	0.032451	0.031019
	ES	0.696897	0.929907	0.968251	0.983365	0.989994	0.983334	1.004903	0.967749	0.99697	1.008887	0.987404	1.016155
	95% conf int	0.024867	0.0286	0.029898	0.030349	0.030602	0.031263	0.032074	0.031358	0.032504	0.031552	0.030476	0.029829
	ES-Bag	0.687611	0.921139	0.958993	0.972453	0.978421	0.971763	0.993294	0.957908	0.987373	0.9983	0.976289	1.004575
	95% conf int	0.024494	0.028134	0.029349	0.029819	0.030061	0.030695	0.031567	0.030883	0.032055	0.031057	0.02993	0.029299
	HMM	1.199831	1.193249	1.192866	1.197998	1.199487	1.186964	1.190484	1.155541	1.15712	1.155621	1.159845	1.153679
	95% conf int	0.025102	0.025426	0.025683	0.025829	0.026044	0.026374	0.026628	0.026495	0.026767	0.027076	0.027252	0.027552
5.56 mm	Naïve	0.685084	0.730032	0.91887	0.761726	0.666737	0.663463	0.754879	0.916979	1.053807	0.879432	0.888271	0.900865
	95% conf int	0.02061	0.024087	0.027582	0.019542	0.017402	0.01614	0.021803	0.024742	0.052025	0.029745	0.030541	0.027098
	ARIMA	0.660676	0.945495	1.032263	1.048454	1.074702	1.090566	1.111598	1.12676	1.138522	1.152028	1.164401	1.187474
	95% conf int	0.018248	0.023324	0.025511	0.027221	0.028819	0.029284	0.029488	0.029795	0.029856	0.029651	0.029749	0.029723
	ARIMA-Bag	0.641491	0.917448	0.997884	1.013815	1.035595	1.05523	1.074705	1.087	1.098853	1.111569	1.123544	1.145198
	95% conf int	0.017542	0.022423	0.024417	0.026094	0.02755	0.028133	0.028439	0.028682	0.028737	0.02853	0.028591	0.028602
	ES	0.602891	0.811088	0.860615	0.834889	0.841989	0.88492	0.937235	0.965586	0.95091	0.91321	0.91482	0.94415
	95% conf int	0.015877	0.019115	0.019577	0.019993	0.021163	0.02233	0.023819	0.026342	0.026398	0.024844	0.025413	0.02581
	ES-Bag	0.59432	0.797591	0.845935	0.821727	0.828661	0.870329	0.921782	0.948491	0.934703	0.896338	0.897672	0.926316
	95% conf int	0.015536	0.018686	0.019164	0.019585	0.020747	0.021894	0.023392	0.0258	0.025862	0.024295	0.024805	0.025199
	HMM	1.010883	1.031191	1.045094	1.052308	1.065176	1.071116	1.087407	1.104285	1.121385	1.130599	1.146731	1.162409
	95% conf int	0.027271	0.02749	0.027354	0.0276	0.027676	0.027828	0.027987	0.027933	0.027914	0.027952	0.027957	0.027951
7.62 mm	Naïve	0.64485	0.791512	0.716327	0.827633	0.955479	0.980784	0.826009	0.806988	0.973244	0.980722	0.993066	1.099306
	95% conf int	0.019749	0.022914	0.020755	0.025439	0.035384	0.032717	0.026257	0.017405	0.033834	0.035595	0.034208	0.046161
	ARIMA	0.65311	0.970975	1.032612	1.104537	1.186451	1.203818	1.175261	1.253146	1.351684	1.397563	1.395253	1.436922
	95% conf int	0.019677	0.032369	0.035635	0.039856	0.042269	0.041621	0.036781	0.044744	0.056086	0.054872	0.055893	0.05553
	ARIMA-Bag	0.636934	0.941941	1.002449	1.069755	1.143151	1.159489	1.13174	1.203199	1.293884	1.339968	1.342511	1.381563
	95% conf int	0.019201	0.031202	0.034391	0.038425	0.040474	0.039802	0.035208	0.042602	0.053321	0.051913	0.053277	0.053202
	ES	0.596832	0.897308	0.937133	1.013117	1.051443	1.038457	0.983382	1.039158	1.089571	1.133189	1.136383	1.197767
	95% conf int	0.018267	0.030084	0.033099	0.037517	0.038443	0.035931	0.031563	0.038141	0.046394	0.042756	0.045041	0.045837
	ES-Bag	0.593818	0.890009	0.930016	1.004329	1.041302	1.029331	0.975075	1.029511	1.079287	1.122417	1.124692	1.185305
	95% conf int	0.018183	0.029842	0.0328	0.037146	0.038015	0.035594	0.031343	0.037757	0.045875	0.042317	0.044442	0.045225
	HMM	1.443318	1.471256	1.505115	1.503896	1.526707	1.543089	1.577424	1.590707	1.615197	1.634982	1.661169	1.682436
	95% conf int	0.04478	0.045688	0.048318	0.046496	0.045704	0.04562	0.046645	0.044912	0.045376	0.045571	0.045397	0.045362
All MG ammunition	Naïve	0.524522	0.560125	0.62086	0.627117	0.651303	0.663187	0.656837	0.661134	0.724411	0.698633	0.717207	0.76185
	95% conf int	0.015351	0.01658	0.017148	0.018353	0.017179	0.017744	0.016077	0.01582	0.02468	0.022153	0.028014	0.028555
	ARIMA	0.469018	0.61602	0.670802	0.697381	0.73065	0.760361	0.787883	0.821979	0.859763	0.87299	0.873161	0.913349
	95% conf int	0.011151	0.017095	0.020698	0.021554	0.022358	0.023599	0.023107	0.02616	0.028179	0.028549	0.027837	0.027333
	ARIMA-Bag	0.457522	0.601679	0.655323	0.680965	0.710835	0.741042	0.768137	0.798582	0.83478	0.847793	0.846354	0.884723
	95% conf int	0.010828	0.016556	0.02007	0.020907	0.021646	0.022825	0.022319	0.025098	0.027132	0.027437	0.026632	0.026165
	ES	0.453607	0.595017	0.638997	0.664325	0.680171	0.69058	0.697344	0.705392	0.724085	0.731432	0.719401	0.760712
	95% conf int	0.011063	0.016627	0.020083	0.020469	0.020883	0.021566	0.0206	0.022939	0.024641	0.024681	0.023249	0.022511
	ES-Bag	0.44346	0.581696	0.624652	0.650186	0.664946	0.674545	0.681927	0.689084	0.707233	0.713979	0.701238	0.741998
	95% conf int	0.010739	0.016079	0.019387	0.019827	0.020205	0.02085	0.01995	0.022226	0.023861	0.023871	0.022461	0.021781
	HMM	0.932548	0.947014	0.953294	0.965462	0.972719	0.984336	0.997905	1.012647	1.021854	1.026259	1.037868	1.047565
	95% conf int	0.022817	0.022784	0.022613	0.022564	0.022803	0.023245	0.02342	0.023423	0.0235	0.023643	0.02358	0.023515

Table 5: Mean Absolute Percentage Error and 95% confidence intervals for forecasting models on 100 simulated surrogate series. Models made weekly forecasts for the quantity of machine gun ammunition demanded in the entire country of Afghanistan. Bagged models improved performance over their respective base models, and did so in a statistically significant manner.

	Weeks Ahead:	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Fifty Caliber	Naïve	0.671476	0.674712	0.741355	0.77514	0.701665	0.67224	0.683407	0.675777	0.65264	0.67143	0.773625
	95% conf int	0.024755	0.018887	0.020441	0.024577	0.024864	0.018486	0.016027	0.016281	0.017508	0.024032	0.039266
	ARIMA	0.609111	0.730676	0.76252	0.771415	0.780553	0.805715	0.843508	0.864065	0.907156	0.911486	0.899294
	95% conf int	0.015242	0.019072	0.021251	0.022788	0.02254	0.02174	0.021757	0.023238	0.023795	0.023009	0.023432
	ARIMA-Bag	0.601839	0.724876	0.753632	0.758048	0.769175	0.797021	0.835969	0.854951	0.896775	0.90088	0.891203
	95% conf int	0.014807	0.018662	0.020734	0.022226	0.021891	0.02112	0.021082	0.022407	0.023048	0.022339	0.022757
	ES	0.595796	0.706164	0.727162	0.729636	0.736338	0.756059	0.784605	0.79267	0.833835	0.850212	0.854938
	95% conf int	0.014838	0.01851	0.020611	0.022048	0.021692	0.021128	0.020817	0.0208	0.021219	0.020951	0.021805
	ES-Bag	0.58667	0.697317	0.716797	0.718611	0.724954	0.744487	0.773508	0.782935	0.823818	0.839256	0.8435
	95% conf int	0.014525	0.01812	0.020175	0.021563	0.021179	0.020612	0.020357	0.020359	0.020826	0.020527	0.021324
	HMM	1.009322	1.004522	1.010481	1.012206	1.018249	1.02453	1.030029	1.019882	1.019777	1.019898	1.019176
	95% conf int	0.020407	0.020693	0.020824	0.021039	0.021127	0.021252	0.021433	0.021784	0.02207	0.022341	0.022601
5.56 mm	Naïve	0.573834	0.696415	0.688175	0.588472	0.535151	0.604173	0.720257	0.830213	0.798484	0.768231	0.784799
	95% conf int	0.018614	0.022738	0.017842	0.01364	0.012177	0.016021	0.021079	0.026439	0.028971	0.026081	0.02561
	ARIMA	0.57908	0.764689	0.80762	0.839284	0.854872	0.87144	0.883033	0.885066	0.886759	0.888846	0.897966
	95% conf int	0.015092	0.019853	0.022	0.023644	0.024136	0.024271	0.024394	0.024626	0.024626	0.024609	0.024597
	ARIMA-Bag	0.558249	0.736146	0.777291	0.806075	0.821378	0.837411	0.847868	0.849789	0.851349	0.852361	0.861151
	95% conf int	0.014496	0.018958	0.020943	0.022508	0.023221	0.023482	0.023597	0.023767	0.023756	0.023739	0.023719
	ES	0.507596	0.631415	0.641389	0.652156	0.676098	0.734285	0.771487	0.77019	0.746482	0.736085	0.741293
	95% conf int	0.012477	0.014891	0.015388	0.016093	0.016992	0.017727	0.018928	0.020822	0.021023	0.020741	0.021167
	ES-Bag	0.498449	0.620038	0.630131	0.64082	0.664001	0.720128	0.756659	0.755827	0.73159	0.719794	0.724683
	95% conf int	0.012204	0.014544	0.015057	0.015759	0.016662	0.017451	0.018635	0.020422	0.020589	0.020292	0.020674
	HMM	0.793584	0.803073	0.806864	0.816077	0.826817	0.838028	0.847512	0.858826	0.862187	0.866369	0.874457
	95% conf int	0.018842	0.018831	0.018958	0.019034	0.019088	0.019095	0.019086	0.019075	0.01916	0.019264	0.019302
7.62 mm	Naïve	0.577163	0.611996	0.631385	0.722366	0.760641	0.76282	0.675419	0.702798	0.766471	0.757388	0.800981
	95% conf int	0.014492	0.013784	0.017627	0.022736	0.023873	0.023103	0.016967	0.015534	0.019779	0.019129	0.023832
	ARIMA	0.596316	0.752162	0.816516	0.888847	0.930746	0.92158	0.935899	0.976534	1.048605	1.085716	1.121074
	95% conf int	0.015849	0.023728	0.027505	0.029679	0.03086	0.027904	0.026767	0.032196	0.035265	0.033889	0.032885
	ARIMA-Bag	0.580273	0.73006	0.786583	0.854393	0.892439	0.879994	0.892378	0.929832	1.000414	1.039315	1.07264
	95% conf int	0.015355	0.022943	0.026541	0.028537	0.029671	0.026869	0.025638	0.030726	0.03356	0.032206	0.031463
	ES	0.564923	0.707329	0.749207	0.799949	0.821525	0.785391	0.772297	0.785812	0.840895	0.881543	0.917303
	95% conf int	0.014972	0.022664	0.026238	0.027729	0.028594	0.025469	0.024333	0.028247	0.030621	0.028702	0.028008
	ES-Bag	0.560204	0.701644	0.74198	0.791335	0.813172	0.777658	0.764246	0.77753	0.832301	0.872578	0.907542
	95% conf int	0.014861	0.02247	0.025971	0.027435	0.028277	0.025255	0.024121	0.027934	0.030263	0.028373	0.027697
	HMM	1.131846	1.152241	1.160404	1.180016	1.203116	1.22282	1.24251	1.256534	1.270899	1.286367	1.303739
	95% conf int	0.027207	0.027868	0.027773	0.02743	0.027302	0.027356	0.027111	0.026777	0.0269	0.026898	0.026622
All MG ammunition	Naïve	0.473212	0.522274	0.539164	0.554909	0.569547	0.590333	0.586625	0.609146	0.608372	0.603406	0.629081
	95% conf int	0.013805	0.012823	0.015018	0.015809	0.015041	0.014922	0.013764	0.014342	0.019144	0.017575	0.02297
	ARIMA	0.431677	0.520358	0.551613	0.589499	0.626955	0.656561	0.689842	0.714722	0.752851	0.774197	0.79969
	95% conf int	0.009946	0.015496	0.017968	0.018827	0.019167	0.018809	0.019312	0.021474	0.022104	0.021528	0.021929
	ARIMA-Bag	0.421098	0.507794	0.537696	0.572773	0.608137	0.637411	0.667895	0.691263	0.728594	0.749365	0.772748
	95% conf int	0.009558	0.014989	0.017386	0.018176	0.018496	0.018147	0.018584	0.020621	0.021246	0.020594	0.020913
	ES	0.422358	0.502474	0.528679	0.553105	0.571769	0.582876	0.587754	0.58922	0.616862	0.630771	0.650375
	95% conf int	0.009337	0.014684	0.016981	0.017434	0.017498	0.016719	0.016901	0.018757	0.019135	0.018208	0.018256
	ES-Bag	0.412199	0.4899	0.515073	0.538656	0.555818	0.567005	0.571778	0.572089	0.599625	0.613803	0.631932
	95% conf int	0.008979	0.014165	0.016421	0.016867	0.016921	0.016182	0.016389	0.018196	0.018557	0.017611	0.017662
	HMM	0.800178	0.807902	0.814183	0.822331	0.832797	0.846914	0.857451	0.866814	0.869966	0.873549	0.882262
	95% conf int	0.018103	0.018066	0.018017	0.018048	0.018207	0.018383	0.018376	0.018401	0.018571	0.018658	0.018634

Table 6: Mean Absolute Percentage Error and 95% confidence intervals for forecasting models on 100 simulated surrogate series. Models made weekly forecasts for the quantity of machine gun ammunition demanded in the entire country of Afghanistan and error was calculated in 2-week periods.

	Typical ARIMA Models			Typical Exponential Smoothing Models			
Series	Autoregressive Order	differencing order	Moving Average Order	Error Component	Trend Component	Seasonal Component	Typical Alpha
.50 caliber	0	1	1	Additive	None	None	0.18
5.56 mm	1	1	1	Additive	None	None	0.23
7.62 mm	2	1	2	Additive	None	None	0.31
All MG	0	1	1	Additive	None	None	0.29

Table 7: Typical Parameter values of time series models by series.

Method	MAPE	95% confidence interval
Generic Unit	64.30%	+/- 5.1%
OPLOG Planner	96.60%	+/- 0.5%

Table 8: Mean Absolute Percentage Error for unit based estimation models on all machine gun series. Models made weekly estimates for the quantity of machine gun ammunition demanded in the entire country of Afghanistan based on expected units present reported in order of battle documents.

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Appendix 1: Results of Monthly Analysis

	Naïve Monthly	2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	22225182	19031855	16276288	27271852	205.60%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	22225182	19031855	16276288	27271852	29600995	14031575	19303001	172.53%
	6 months ahead	NA	NA	NA	NA	NA	22225182	19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	85.06%
	3 months ahead	NA	NA	22225182	19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	22.54%
	1 month ahead	22225182	19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7.22%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	186.75%	261.26%	130.15%	256.06%	**205.60%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	213.90%	128.48%	47.28%	251.86%	461.88%	98.41%	152.02%	**172.53%**
	6 months ahead	NA	NA	NA	NA	NA	15.14%	65.96%	53.42%	285.18%	255.36%	26.97%	149.05%	117.68%	50.02%	7.56%	**86.27%**
	3 months ahead	NA	NA	18.51%	35.71%	16.00%	41.28%	158.12%	32.26%	172.63%	37.67%	4.00%	8.65%	58.12%	56.26%	1.19%	**42.64%**
	1 month ahead	16.78%	16.93%	40.32%	7.87%	110.96%	27.31%	68.32%	8.09%	49.84%	15.00%	24.62%	42.58%	47.12%	25.51%	7.67%	**31.95%**

Forecast notes: Next month's demand is assumed to be equal to current month's demand

	Naïve Yearly	2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	22225182	19031855	16276288	27271852	205.60%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	10862614	41523427	12170012	22225182	19031855	16276288	27271852	175.52%
	6 months ahead	NA	NA	NA	NA	NA	26542618	22365688	28228640	10862614	41523427	12170012	22225182	19031855	16276288	27271852	136.94%
	3 months ahead	NA	NA	30245886	17283950	17832804	26542618	22365688	28228640	10862614	41523427	12170012	22225182	19031855	16276288	27271852	75.30%
	1 month ahead	18413117	18981397	30245886	17283950	17832804	26542618	22365688	28228640	10862614	41523427	12170012	22225182	19031855	16276288	27271852	63.16%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	186.75%	261.26%	130.15%	256.06%	**205.60%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	53.42%	398.49%	10.13%	186.75%	261.26%	130.15%	256.06%	**175.52%**
	6 months ahead	NA	NA	NA	NA	NA	37.51%	95.03%	166.08%	53.42%	398.49%	10.13%	186.75%	261.26%	130.15%	256.06%	**136.94%**
	3 months ahead	NA	NA	10.91%	41.61%	27.09%	37.51%	95.03%	166.08%	53.42%	398.49%	10.13%	186.75%	261.26%	130.15%	256.06%	**90.09%**
	1 month ahead	3.25%	16.62%	10.91%	41.61%	27.09%	37.51%	95.03%	166.08%	53.42%	398.49%	10.13%	186.75%	261.26%	130.15%	256.06%	**75.98%**

Forecast notes: Next month's demand is assumed to be equal to next month's demand one year ago

	OPLOG Planner	2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	312188	312188	281976	312188	95.61%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	302118	312188	304332	312188	312188	281976	312188	96.06%
	6 months ahead	NA	NA	NA	NA	NA	338655	349944	349944	302118	312188	304332	312188	312188	281976	312188	96.68%
	3 months ahead	NA	NA	349944	338655	349944	338655	349944	349944	302118	312188	304332	312188	312188	281976	312188	97.47%
	1 month ahead	349944	327367	349944	338655	349944	338655	349944	349944	302118	312188	304332	312188	312188	281976	312188	97.58%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	95.97%	94.07%	96.01%	95.92%	**95.61%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	95.73%	96.25%	97.25%	95.97%	94.07%	96.01%	95.92%	**96.06%**
	6 months ahead	NA	NA	NA	NA	NA	98.25%	96.95%	96.70%	95.73%	96.25%	97.25%	95.97%	94.07%	96.01%	95.92%	**96.68%**
	3 months ahead	NA	NA	98.72%	98.86%	97.51%	98.25%	96.95%	96.70%	95.73%	96.25%	97.25%	95.97%	94.07%	96.01%	95.92%	**97.47%**
	1 month ahead	98.16%	97.99%	98.72%	98.86%	97.51%	98.25%	96.95%	96.70%	95.73%	96.25%	97.25%	95.97%	94.07%	96.01%	95.92%	**97.58%**

Forecast notes: Monthly demand is estimated by OPLOG planner assuming unit information is known 12 months in the future

Table 1: Forecast results of Naive methods

Holt-Winters (Uni)		2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	27823501	29432643	29015170	36479414	342.34%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	14894428	36661547	37385310	24698863	29432643	29015170	39592640	290.47%
	6 months ahead	NA	NA	NA	NA	NA	31205987	17881741	16096696	11595733	36661547	37385310	27812089	25874929	24726836	34328518	175.72%
	3 months ahead	NA	NA	34097141	20521361	21527658	27977466	17881741	16096696	14708959	33103833	33096976	22547966	7722737	2970106	12548923	59.04%
	1 month ahead	23769866	23314140	33891611	17408135	21527658	31090692	17881741	12538982	10420626	31397424	16241882	7157298	5739661	3073668	13649893	33.35%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	258.98%	458.69%	310.28%	376.28%	**342.34%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	110.37%	340.12%	238.30%	218.66%	458.69%	310.28%	416.92%	**290.47%**
	6 months ahead	NA	NA	NA	NA	NA	61.66%	55.93%	51.72%	63.78%	340.12%	238.30%	258.83%	391.16%	249.64%	348.20%	**175.72%**
	3 months ahead	NA	NA	25.03%	30.67%	53.42%	44.94%	55.93%	51.72%	107.75%	297.41%	199.49%	190.91%	46.59%	58.00%	63.84%	**74.88%**
	1 month ahead	24.90%	43.24%	24.27%	41.19%	53.42%	61.07%	55.93%	18.19%	47.18%	276.93%	46.97%	7.66%	8.95%	56.54%	78.21%	**49.98%**

Forecast notes: Univariate Holt Winters from R package "stats", additive seasonality, univariate(only machine gun totals)

Holt-Winters (Multi)		2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	24972745	22638683	19530765	25403909	233.50%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	19728041	28222460	16281223	19424334	27740640	24035998	24389018	194.81%
	6 months ahead	NA	NA	NA	NA	NA	27189290	19060056	16958922	15081512	29340924	20757415	19937898	18269658	17232918	20002954	113.23%
	3 months ahead	NA	NA	27744372	19873595	17180497	22664990	24080148	20733430	16822786	23411830	13682324	13600165	10785510	12001215	12414594	41.14%
	1 month ahead	25072737	24253758	20961790	18840219	24275789	23310736	18542029	16552730	10571929	21630653	9222865	10321386	10535173	9028742	9399519	25.13%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	222.20%	329.73%	176.17%	231.68%	**233.50%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	178.63%	238.81%	47.33%	150.61%	426.57%	239.87%	218.43%	**194.81%**
	6 months ahead	NA	NA	NA	NA	NA	40.86%	66.20%	59.85%	113.01%	252.24%	87.83%	157.24%	246.79%	143.67%	161.16%	**113.23%**
	3 months ahead	NA	NA	1.73%	32.86%	22.44%	17.42%	109.98%	95.43%	137.60%	181.06%	23.81%	75.47%	104.73%	69.70%	62.09%	**52.83%**
	1 month ahead	31.74%	49.01%	23.14%	36.35%	73.01%	20.76%	61.69%	56.02%	49.32%	159.68%	16.54%	33.17%	99.98%	27.67%	22.72%	**43.86%**

Forecast notes: Univariate Holt Winters from R package "stats", additive seasonality, multivariate(US BOG, Attacks, and machine gun totals)

Table 2: Forecast results of Exponential Smoothing methods

Hidden Markov (2)		2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	16881016	17013181	21913503	19656965	171.94%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	16584173	16866718	19996357	21174144	20949549	20799643	20479068	152.44%
	6 months ahead	NA	NA	NA	NA	NA	17484485	17738411	21228682	20690972	21984466	21889201	21739622	17602416	20976555	17602715	108.11%
	3 months ahead	NA	NA	18686847	18694674	18242976	20566486	19490445	18763227	19175837	18956462	19059210	18407472	18114151	18175036	18152012	46.84%
	1 month ahead	15819846	16132158	16133506	20646322	17150262	16735285	16793141	16024971	16173038	16066869	15901716	21729802	21718954	21715704	21709341	34.02%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	117.80%	222.94%	209.86%	156.64%	**171.94%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	134.23%	102.49%	80.95%	173.19%	297.66%	194.11%	167.38%	**152.44%**
	6 months ahead	NA	NA	NA	NA	NA	9.42%	54.68%	100.10%	192.23%	163.92%	98.08%	180.48%	234.13%	196.61%	129.82%	**111.92%**
	3 months ahead	NA	NA	31.48%	36.84%	30.01%	6.55%	69.96%	76.86%	170.84%	127.57%	72.47%	137.49%	243.84%	157.00%	136.99%	**70.25%**
	1 month ahead	16.88%	0.89%	40.84%	30.25%	22.23%	13.30%	46.44%	51.05%	128.42%	92.88%	43.89%	180.36%	312.27%	207.06%	183.44%	**59.80%**

Forecast notes: A 2-state HMM from R package "hsmm"

Hidden Markov (3)		2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	11412864	11555484	9917502	10038573	54.68%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	10328984	9883415	9269950	11448803	12011193	10236476	10045175	35.07%
	6 months ahead	NA	NA	NA	NA	NA	11707401	10196537	10851110	10504205	10147348	9563492	11522744	10786983	10703273	11372872	12.31%
	3 months ahead	NA	NA	10472746	10852925	10605624	11910027	10370588	11022240	10456963	10843615	10098539	11782817	11887532	10830216	10975526	14.65%
	1 month ahead	10893811	10989346	10475798	10968839	11208574	11642008	10325375	10182507	11110369	11388126	11093779	10865285	12140339	11387829	11108551	17.85%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	47.25%	119.35%	40.23%	31.06%	**54.68%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	45.88%	18.65%	16.12%	47.71%	128.00%	44.74%	31.15%	**41.64%**
	6 months ahead	NA	NA	NA	NA	NA	39.35%	11.09%	2.28%	48.36%	21.82%	13.46%	48.67%	104.76%	51.34%	48.49%	**33.97%**
	3 months ahead	NA	NA	61.60%	63.34%	24.42%	38.30%	9.57%	3.89%	47.69%	30.18%	8.62%	52.02%	125.65%	53.14%	43.30%	**43.51%**
	1 month ahead	42.76%	32.48%	61.59%	62.94%	20.12%	39.69%	9.96%	4.02%	56.92%	36.71%	0.39%	40.18%	130.45%	61.02%	45.03%	**42.51%**

Forecast notes: A 3-state HMM from R package "hsmm"

Hidden Markov (10)		2012												2013			
	Forecast Horizon	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total Error
Forecasts	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	17647291	15647360	16941797	14720797	134.08%
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	17124353	16583696	16587841	17217479	16733882	16373580	15435425	114.08%
	6 months ahead	NA	NA	NA	NA	NA	15109462	15340521	16546023	17321770	16553578	17548484	17802980	16537930	16668631	15559442	72.60%
	3 months ahead	NA	NA	16709239	16762307	15987683	15369886	16492891	18230957	17323263	17523169	17655187	16178058	15490268	16543877	14874318	29.22%
	1 month ahead	16320142	16309346	17005735	16473519	17414757	18497718	16695202	18387460	16056915	15683319	16146586	15623119	14693615	11887590	12718483	18.88%
Actual demand:		19031855	16276288	27271852	29600995	14031575	19303001	11467858	10609238	7080256	8329839	11050957	7750784	5268172	7072107	7659262	**Avg Error**
% Error	12 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	127.68%	197.02%	139.56%	92.20%	**134.08%**
	9 months ahead	NA	NA	NA	NA	NA	NA	NA	NA	141.86%	99.09%	50.10%	122.14%	217.64%	131.52%	101.53%	**114.08%**
	6 months ahead	NA	NA	NA	NA	NA	21.72%	33.77%	55.96%	144.65%	98.73%	58.80%	129.69%	213.92%	135.70%	103.15%	**81.37%**
	3 months ahead	NA	NA	38.73%	43.37%	13.94%	20.38%	43.82%	71.84%	144.67%	110.37%	59.76%	108.73%	194.03%	133.93%	94.20%	**62.05%**
	1 month ahead	14.25%	0.20%	37.64%	44.35%	24.11%	4.17%	45.58%	73.32%	126.78%	88.28%	46.11%	101.57%	178.91%	68.09%	66.05%	**45.55%**

Forecast notes: A 10-state HMM from R package "hsmm"

Table 3: Forecast results of Hidden Markov Models

Appendix 2: Results of Weekly Analysis

	Weeks Ahead:		1	2	3	4	5	6	7	8	9	10	11	12
All MG ammunition	Naïve		0.524522	0.560125	0.62086	0.627117	0.651303	0.663187	0.656837	0.661134	0.724411	0.698633	0.717207	0.76185
		95% conf int	0.015351	0.01658	0.017148	0.018353	0.017179	0.017744	0.016077	0.01582	0.02468	0.022153	0.028014	0.028555
	Naïve Bottom-Up		0.523138	0.560483	0.620708	0.626649	0.652244	0.663436	0.657664	0.66231	0.72639	0.698592	0.717344	0.762473
		95% conf int	0.015417	0.016685	0.017204	0.018461	0.017176	0.017742	0.016073	0.015882	0.024711	0.022166	0.028186	0.028621
	ARIMA		0.469018	0.61602	0.670802	0.697381	0.73065	0.760361	0.787883	0.821979	0.859763	0.87299	0.873161	0.913349
		95% conf int	0.011151	0.017095	0.020698	0.021554	0.022358	0.023599	0.023107	0.02616	0.028179	0.028549	0.027837	0.027333
	ARIMA-Bag		0.457522	0.601679	0.655323	0.680965	0.710835	0.741042	0.768137	0.798582	0.83478	0.847793	0.846354	0.884723
		95% conf int	0.010828	0.016556	0.02007	0.020907	0.021646	0.022825	0.022319	0.025098	0.027132	0.027437	0.026632	0.026165
	ARIMA Bottom-Up		0.495662	0.678926	0.740086	0.775009	0.80662	0.819699	0.831754	0.842878	0.867894	0.875266	0.870897	0.900462
		95% conf int	0.011905	0.018092	0.021331	0.022373	0.023099	0.023451	0.022667	0.02479	0.026838	0.026707	0.026094	0.025695
	ARIMA-Bag Bottom-Up		0.484101	0.661072	0.719009	0.751094	0.781513	0.796061	0.80764	0.818767	0.842325	0.85017	0.845728	0.873265
		95% conf int	0.011627	0.017626	0.020696	0.021578	0.022232	0.022654	0.021896	0.023897	0.025883	0.025785	0.025198	0.024896
	ES		0.453607	0.595017	0.638997	0.664325	0.680171	0.69058	0.697344	0.705392	0.724085	0.731432	0.719401	0.760712
		95% conf int	0.011063	0.016627	0.020083	0.020469	0.020883	0.021566	0.0206	0.022939	0.024641	0.024681	0.023249	0.022511
	ES-Bag		0.44346	0.581696	0.624652	0.650186	0.664946	0.674545	0.681927	0.689084	0.707233	0.713979	0.701238	0.741998
		95% conf int	0.010739	0.016079	0.019387	0.019827	0.020205	0.02085	0.01995	0.022226	0.023861	0.023871	0.022461	0.021781
	ES Bottom-Up		0.464443	0.624829	0.661309	0.68904	0.708145	0.721719	0.728427	0.732346	0.748583	0.761266	0.755378	0.796964
		95% conf int	0.011343	0.017367	0.020316	0.020801	0.021001	0.021052	0.020181	0.022106	0.023591	0.023428	0.022409	0.022124
	ES-Bag Bottom-Up		0.459158	0.617562	0.65343	0.68058	0.699204	0.712317	0.718795	0.722708	0.738614	0.750846	0.744522	0.785084
		95% conf int	0.011159	0.017045	0.019913	0.020383	0.020575	0.020672	0.019865	0.021774	0.023248	0.023051	0.022015	0.021738
	HMM		0.932548	0.947014	0.953294	0.965462	0.972719	0.984336	0.997905	1.012647	1.021854	1.026259	1.037868	1.047565
		95% conf int	0.022817	0.022784	0.022613	0.022564	0.022803	0.023245	0.02342	0.023423	0.0235	0.023643	0.02358	0.023515
	HMM Bottom-Up		1.001966	1.010014	1.017708	1.025355	1.034847	1.033997	1.045045	1.05325	1.064239	1.068964	1.081913	1.09184
		95% conf int	0.023843	0.023805	0.023981	0.024115	0.023886	0.023966	0.024034	0.024069	0.024128	0.024314	0.024268	0.024123

Table 1: Mean Absolute Percentage Error and 95% confidence intervals for forecasting models on 100 simulated surrogate series. Models made weekly forecasts for the quantity of machine gun ammunition demanded in the entire country of Afghanistan. Bottom-up models used the sum of forecasts on the constituent (.50 caliber, 5.56mm, and 7.62mm) series.

	Weeks Ahead:	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
All MG ammunition	Naïve	0.473212	0.522274	0.539164	0.554909	0.569547	0.590333	0.586625	0.609146	0.608372	0.603406	0.629081
	95% conf int	0.013805	0.012823	0.015018	0.015809	0.015041	0.014922	0.013764	0.014342	0.019144	0.017575	0.02297
	Naïve Bottom-Up	0.471476	0.521946	0.537899	0.554252	0.569685	0.590396	0.587308	0.611015	0.60844	0.603286	0.630447
	95% conf int	0.013887	0.012862	0.015102	0.015841	0.015046	0.014942	0.013809	0.014374	0.019199	0.017593	0.0231
	ARIMA	0.431677	0.520358	0.551613	0.589499	0.626955	0.656561	0.689842	0.714722	0.752851	0.774197	0.79969
	95% conf int	0.009946	0.015496	0.017968	0.018827	0.019167	0.018809	0.019312	0.021474	0.022104	0.021528	0.021929
	ARIMA-Bag	0.421098	0.507794	0.537696	0.572773	0.608137	0.637411	0.667895	0.691263	0.728594	0.749365	0.772748
	95% conf int	0.009558	0.014989	0.017386	0.018176	0.018496	0.018147	0.018584	0.020621	0.021246	0.020594	0.020913
	ARIMA Bottom-Up	0.454711	0.582254	0.627644	0.667868	0.693719	0.705199	0.713299	0.727076	0.754734	0.767577	0.784168
	95% conf int	0.010866	0.016263	0.018583	0.019642	0.019697	0.018813	0.018896	0.020594	0.021351	0.02099	0.021192
	ARIMA-Bag Bottom-Up	0.442557	0.565943	0.60627	0.642696	0.667584	0.679947	0.688738	0.70249	0.729524	0.742787	0.758743
	95% conf int	0.010508	0.015723	0.017894	0.01887	0.018963	0.018139	0.018194	0.019808	0.020552	0.020215	0.020452
	ES	0.422358	0.502474	0.528679	0.553105	0.571769	0.582876	0.587754	0.58922	0.616862	0.630771	0.650375
	95% conf int	0.009337	0.014684	0.016981	0.017434	0.017498	0.016719	0.016901	0.018757	0.019135	0.018208	0.018256
	ES-Bag	0.412199	0.4899	0.515073	0.538656	0.555818	0.567005	0.571778	0.572089	0.599625	0.613803	0.631932
	95% conf int	0.008979	0.014165	0.016421	0.016867	0.016921	0.016182	0.016389	0.018196	0.018557	0.017611	0.017662
	ES Bottom-Up	0.431754	0.523626	0.553589	0.581666	0.600374	0.609669	0.612564	0.617602	0.646399	0.664012	0.682022
	95% conf int	0.010188	0.015476	0.017393	0.017897	0.01772	0.016752	0.016744	0.018136	0.018624	0.018062	0.01826
	ES-Bag Bottom-Up	0.425421	0.515817	0.545107	0.572179	0.590042	0.599598	0.601875	0.606576	0.634737	0.652436	0.670057
	95% conf int	0.009983	0.015171	0.01704	0.017532	0.017377	0.016456	0.016488	0.017867	0.01834	0.017743	0.017911
	HMM	0.800178	0.807902	0.814183	0.822331	0.832797	0.846914	0.857451	0.866814	0.869966	0.873549	0.882262
	95% conf int	0.018103	0.018066	0.018017	0.018048	0.018207	0.018383	0.018376	0.018401	0.018571	0.018658	0.018634
	HMM Bottom-Up	0.855745	0.861523	0.866642	0.876145	0.8866	0.895516	0.905965	0.916061	0.920054	0.924275	0.933666
	95% conf int	0.01891	0.019102	0.019288	0.019272	0.019152	0.019115	0.019108	0.019075	0.019244	0.019357	0.019295

Table 2: Mean Absolute Percentage Error and 95% confidence intervals for forecasting models on 100 simulated surrogate series. Models made weekly forecasts for the quantity of machine gun ammunition demanded in the entire country of Afghanistan. Bottom-up models used the sum of forecasts on the constituent (.50 caliber, 5.56mm, and 7.62mm) series and error was calculated in 2-week periods.

Generic Unit Estimates based on Order of Battle inputs												
	Estimate				Actual				Error			
Week	Fifty	556	762	Total	Fifty	556	762	Total	Fifty	556	762	Total
106	136272	279591	302181	718044	659585	1469047	1609537	3738169	0.793397	0.809679	0.812256	0.807916
107	136272	279591	302181	718044	773981	1002381	994335	2770697	0.823934	0.721073	0.696097	0.740844
108	136272	279591	302181	718044	1657001	683315	1088848	3429164	0.91776	0.590831	0.722476	0.790607
109	136272	279591	302181	718044	2163231	2035311	2418848	6617390	0.937005	0.86263	0.875072	0.891491
110	136272	279591	302181	718044	660435	569351	1083271	2313057	0.793663	0.50893	0.721048	0.689569
111	136272	279591	302181	718044	1927650	1754490	2119114	5801254	0.929307	0.840643	0.857402	0.876226
112	136272	279591	302181	718044	360868	910066	2206120	3477054	0.622377	0.692779	0.863026	0.793491
113	136272	279591	302181	718044	870299	1473236	2772614	5116149	0.843419	0.81022	0.891012	0.859651
114	136272	279591	302181	718044	4151502	3108125	3707239	10966866	0.967175	0.910045	0.918489	0.934526
115	136272	279591	302181	718044	1005110	1458949	1617969	4082028	0.864421	0.808361	0.813234	0.824096
116	136272	279591	302181	718044	818877	1385614	1795487	3999978	0.833587	0.798219	0.8317	0.820488
117	136272	279591	302181	718044	857456	621531	2772913	4251900	0.841074	0.550158	0.891024	0.831124
118	136272	279591	302181	718044	1041912	1303549	3218606	5564067	0.86921	0.785516	0.906114	0.87095
119	136272	279591	302181	718044	2161244	2723276	4178026	9062546	0.936947	0.897333	0.927674	0.920768
120	136272	279591	302181	718044	4102303	3412862	4032630	11547795	0.966782	0.918077	0.925066	0.93782
121	136272	279591	302181	718044	1319776	1480164	2206534	5006474	0.896746	0.811108	0.863052	0.856577
122	136272	279591	302181	718044	1007829	931100	1150522	3089451	0.864787	0.69972	0.737353	0.767582
123	136272	279591	302181	718044	1459365	892604	1087835	3439804	0.906622	0.686769	0.722218	0.791254
124	136272	279591	302181	718044	606240	836371	758874	2201485	0.775218	0.665709	0.601803	0.673837
125	136272	279591	302181	718044	523338	1179833	1284953	2988124	0.73961	0.763025	0.764831	0.759701
126	136272	279591	302181	718044	546290	1613185	1136235	3295710	0.75055	0.826684	0.734051	0.782128
127	136272	279591	302181	718044	1231716	1116923	1932284	4280923	0.889364	0.749677	0.843615	0.832269
128	136272	279591	302181	718044	1714238	2078835	2830725	6623798	0.920506	0.865506	0.89325	0.891596
129	136272	279591	302181	718044	1414426	382220	815317	2611963	0.903656	0.268508	0.62937	0.725094
130	136272	279591	302181	718044	2622345	626901	2164830	5414076	0.948034	0.554011	0.860414	0.867375
131	136272	279591	302181	718044	338136	500519	490560	1329215	0.596991	0.441398	0.384008	0.459798
132	136272	279591	302181	718044	357665	784001	705508	1847174	0.618995	0.643379	0.571683	0.611274
133	136272	279591	302181	718044	1234267	2875816	1919193	6029276	0.889593	0.902779	0.842548	0.880907
134	136272	279591	302181	718044	505918	577817	457572	1541307	0.730644	0.516125	0.339599	0.534133
135	136272	279591	302181	718044	484548	383117	605898	1473563	0.718765	0.27022	0.501268	0.512716
136	136272	279591	302181	718044	617972	414968	706128	1739068	0.779485	0.326235	0.572059	0.58711
137	136272	279591	302181	718044	667245	866587	1406925	2940757	0.795769	0.677365	0.785219	0.75583
138	136272	279591	302181	718044	487812	955467	789914	2233193	0.720646	0.707378	0.617451	0.678468
139	136272	279591	302181	718044	1062354	1020565	945560	3028479	0.871726	0.726043	0.680421	0.762903
140	155639	317766	353646	827051	495470	545890	1195119	2236479	0.685876	0.417894	0.704091	0.6302
141	155639	317766	353646	827051	995038	510073	1021853	2526964	0.843585	0.377019	0.653917	0.67271
142	155639	317766	353646	827051	289288	184108	420110	893506	0.461993	0.725976	0.158206	0.074376
143	155639	317766	353646	827051	256699	618507	418698	1293904	0.393691	0.486237	0.155367	0.36081
144	155639	317766	353646	827051	339151	523633	935955	1798739	0.541092	0.393151	0.622155	0.540205
145	155639	317766	353646	827051	609732	518297	800565	1928594	0.744742	0.386904	0.558254	0.571164
146	155639	317766	353646	827051	588726	433110	499887	1521723	0.735634	0.266316	0.292548	0.456504
147	155639	317766	353646	827051	385238	868670	543970	1797878	0.595993	0.634193	0.34988	0.539985
148	155639	317766	353646	827051	576638	935320	556194	2068152	0.730092	0.66026	0.364168	0.600101
149	155639	317766	353646	827051	365130	866100	419147	1650377	0.573744	0.633107	0.156272	0.498871
150	155639	317766	353646	827051	368484	1171644	1479871	3019999	0.577623	0.728786	0.761029	0.726142
151	155639	317766	353646	827051	577372	367639	1674500	2619511	0.730435	0.135658	0.788805	0.684273
152	155639	317766	353646	827051	865418	826900	1590293	3282611	0.820157	0.615714	0.777622	0.748051
153	155639	317766	353646	827051	633847	622700	752211	2008758	0.754453	0.489696	0.529858	0.588277
154	155639	317766	353646	827051	501814	633266	873779	2008859	0.689847	0.498211	0.595268	0.588298
155	155639	317766	353646	827051	624991	447122	967307	2039420	0.750974	0.289308	0.634401	0.594468
156	155639	317766	353646	827051	466896	263318	324758	1054972	0.666652	0.206777	0.088952	0.216045
157	155639	317766	353646	827051	389439	358302	207448	955189	0.600351	0.113134	0.704745	0.134149
158	155639	317766	353646	827051	237074	291420	499110	1027604	0.3435	0.090406	0.291447	0.195166
159	155639	317766	353646	827051	535731	639480	1060865	2236076	0.709483	0.503087	0.666644	0.630133
160	155639	317766	353646	827051	406619	314101	537995	1258715	0.617236	0.011668	0.342659	0.34294
161	155639	317766	353646	827051	191542	158568	189173	539283	0.187442	1.003973	0.869432	0.533612
162	155639	317766	353646	827051	304557	254606	606926	1166089	0.488966	0.24807	0.417316	0.290748
163	155639	317766	353646	827051	792390	710626	572099	2075115	0.803583	0.552837	0.381845	0.601443
164	155639	317766	353646	827051	1258714	308640	121836	1689190	0.876351	0.029568	1.90264	0.510386
165	155639	317766	353646	827051	939588	435403	766722	2141713	0.834354	0.27018	0.538756	0.613837

Table 3: Results of the Generic Unit estimator for weeks 106 to 170.

OPLOG Planner Estimates based on Order of Battle inputs												
	Estimate				Actual				Error			
Week	Fifty	556	762	Total	Fifty	556	762	Total	Fifty	556	762	Total
106	17050.04	20135.43	32803.31	69988.78	659585	1469047	1609537	3738169	0.97415	0.986294	0.979619	0.981277
107	17050.04	20135.43	32803.31	69988.78	773981	1002381	994335	2770697	0.977971	0.979912	0.96701	0.97474
108	17050.04	20135.43	32803.31	69988.78	1657001	683315	1088848	3429164	0.98971	0.970533	0.969873	0.97959
109	17050.04	20135.43	32803.31	69988.78	2163231	2035311	2418848	6617390	0.992118	0.990107	0.986438	0.989424
110	17050.04	20135.43	32803.31	69988.78	660435	569351	1083271	2313057	0.974184	0.964634	0.969718	0.969742
111	26583.4	31393.95	51144.94	109122.3	1927650	1754490	2119114	5801254	0.986209	0.982107	0.975865	0.98119
112	26583.4	31393.95	51144.94	109122.3	360868	910066	2206120	3477054	0.926335	0.965504	0.976817	0.968616
113	26583.4	31393.95	51144.94	109122.3	870299	1473236	2772614	5116149	0.969455	0.97869	0.981554	0.978671
114	17050.04	20135.43	32803.31	69988.78	4151502	3108125	3707239	10966866	0.995893	0.993522	0.991152	0.993618
115	17050.04	20135.43	32803.31	69988.78	1005110	1458949	1617969	4082028	0.983037	0.986199	0.979726	0.982854
116	17050.04	20135.43	32803.31	69988.78	818877	1385614	1795487	3999978	0.979179	0.985468	0.98173	0.982503
117	17050.04	20135.43	32803.31	69988.78	857456	621531	2772913	4251900	0.980116	0.967603	0.98817	0.983539
118	17050.04	20135.43	32803.31	69988.78	1041912	1303549	3218606	5564067	0.983636	0.984553	0.989808	0.987421
119	20625.05	24357.37	39681.42	84663.85	2161244	2723276	4178026	9062546	0.990457	0.991056	0.990502	0.990658
120	20625.05	24357.37	39681.42	84663.85	4102303	3412862	4032630	11547795	0.994972	0.992863	0.99016	0.992668
121	20625.05	24357.37	39681.42	84663.85	1319776	1480164	2206534	5006474	0.984372	0.983544	0.982016	0.983089
122	20625.05	24357.37	39681.42	84663.85	1007829	931100	1150522	3089451	0.979535	0.97384	0.96551	0.972596
123	17050.04	20135.43	32803.31	69988.78	1459365	892604	1087835	3439804	0.988317	0.977442	0.969845	0.979653
124	17050.04	20135.43	32803.31	69988.78	606240	836371	758874	2201485	0.971876	0.975925	0.956774	0.968208
125	17050.04	20135.43	32803.31	69988.78	523338	1179833	1284953	2988124	0.967421	0.982934	0.974471	0.976578
126	17050.04	20135.43	32803.31	69988.78	546290	1613185	1136235	3295710	0.968789	0.987518	0.97113	0.978764
127	17050.04	20135.43	32803.31	69988.78	1231716	1116923	1932284	4280923	0.986157	0.981972	0.983024	0.983651
128	20625.05	24357.37	39681.42	84663.85	1714238	2078835	2830725	6623798	0.987968	0.988283	0.985982	0.987218
129	20625.05	24357.37	39681.42	84663.85	1414426	382220	815317	2611963	0.985418	0.936274	0.95133	0.967586
130	20625.05	24357.37	39681.42	84663.85	2622345	626901	2164830	5414076	0.992135	0.961146	0.98167	0.984362
131	20625.05	24357.37	39681.42	84663.85	338136	500519	490560	1329215	0.939004	0.951336	0.91911	0.936305
132	21312.56	25169.29	41004.13	87485.97	357665	784001	705508	1847174	0.940412	0.967896	0.94188	0.952638
133	21312.56	25169.29	41004.13	87485.97	1234267	2875816	1919193	6029276	0.982733	0.991248	0.978635	0.98549
134	21312.56	25169.29	41004.13	87485.97	505918	577817	457572	1541307	0.957873	0.956441	0.910388	0.943239
135	21312.56	25169.29	41004.13	87485.97	484548	383117	605898	1473563	0.956016	0.934304	0.932325	0.94063
136	21312.56	25169.29	41004.13	87485.97	617972	414968	706128	1739068	0.965512	0.939346	0.941931	0.949694
137	21312.56	25169.29	41004.13	87485.97	667245	866587	1406925	2940757	0.968059	0.970956	0.970855	0.970251
138	21312.56	25169.29	41004.13	87485.97	487812	955467	789914	2233193	0.95631	0.973658	0.94809	0.960825
139	21312.56	25169.29	41004.13	87485.97	1062354	1020565	945560	3028479	0.979938	0.975338	0.956635	0.971112
140	18760.92	22181.41	34587.05	75529.38	495470	545890	1195119	2236479	0.962135	0.959367	0.97106	0.966228
141	18760.92	22181.41	34587.05	75529.38	995038	510073	1021853	2526964	0.981146	0.956513	0.966153	0.970111
142	18760.92	22181.41	34587.05	75529.38	289288	184108	420110	893506	0.935148	0.87952	0.917671	0.915469
143	18760.92	22181.41	34587.05	75529.38	256699	618507	418698	1293904	0.926915	0.964137	0.917394	0.941627
144	15509.03	18336.63	28591.96	62437.62	339151	523633	935955	1798739	0.954271	0.964982	0.969452	0.965288
145	15509.03	18336.63	28591.96	62437.62	609732	518297	800565	1928594	0.974564	0.964621	0.964285	0.967625
146	15509.03	18336.63	28591.96	62437.62	588726	433110	499887	1521723	0.973657	0.957663	0.942803	0.958969
147	15509.03	18336.63	28591.96	62437.62	385238	868670	543970	1797878	0.959742	0.978891	0.947438	0.965271
148	15509.03	18336.63	28591.96	62437.62	576638	935320	556194	2068152	0.973104	0.980395	0.948594	0.96981
149	19314.62	22181.41	34587.05	76083.08	365130	866100	419147	1650377	0.947102	0.974389	0.917482	0.9539
150	19314.62	22181.41	34587.05	76083.08	368484	1171644	1479871	3019999	0.947584	0.981068	0.976628	0.974807
151	19314.62	22181.41	34587.05	76083.08	577372	367639	1674500	2619511	0.966547	0.939665	0.979345	0.970955
152	19314.62	22181.41	34587.05	76083.08	865418	826900	1590293	3282611	0.977682	0.973175	0.978251	0.976822
153	15509.03	18336.63	28591.96	62437.62	633847	622700	752211	2008758	0.975532	0.970553	0.961989	0.968917
154	15509.03	18336.63	28591.96	62437.62	501814	633266	873779	2008859	0.969094	0.971044	0.967278	0.968919
155	15509.03	18336.63	28591.96	62437.62	624991	447122	967307	2039420	0.975185	0.95899	0.970442	0.969385
156	15509.03	18336.63	28591.96	62437.62	466896	263318	324758	1054972	0.966783	0.930363	0.911959	0.940816
157	15509.03	18336.63	28591.96	62437.62	389439	358302	207448	955189	0.960176	0.948824	0.862173	0.934633
158	15509.03	18336.63	28591.96	62437.62	237074	291420	499110	1027604	0.934581	0.937078	0.942714	0.93924
159	15509.03	18336.63	28591.96	62437.62	535731	639480	1060865	2236076	0.971051	0.971326	0.973048	0.972077
160	15509.03	18336.63	28591.96	62437.62	406619	314101	537995	1258715	0.961859	0.941622	0.946855	0.950396
161	15509.03	18336.63	28591.96	62437.62	191542	158568	189173	539283	0.919031	0.884361	0.848858	0.884221
162	15509.03	18336.63	28591.96	62437.62	304557	254606	606926	1166089	0.949077	0.92798	0.952891	0.946456
163	17510.19	20702.65	32281.25	70494.09	792390	710626	572099	2075115	0.977902	0.970867	0.943574	0.966029
164	17510.19	20702.65	32281.25	70494.09	1258714	308640	121836	1689190	0.986089	0.932923	0.735043	0.958268
165	17510.19	20702.65	32281.25	70494.09	939588	435403	766722	2141713	0.981364	0.952452	0.957897	0.967085

Table 4: Results of the OPLOG Planner estimator for weeks 106 to 170..